Local Supersymmetry

$$[\theta Q, \bar{Q}\bar{\theta}] = 2\theta\sigma_\mu\bar{\theta}p^\mu$$

as space time translation

What happens if $\theta_\alpha = \theta_\alpha(x)$?

→ $\theta(x)\sigma_\mu\bar{\theta}\partial^\mu$ differs from point to point

→ general coordinate transformation

Gauge theories

e.g. $\mathcal{L} = i\bar{\psi}\partial_\mu\gamma^\mu\psi$

$$\psi \to e^{-i\epsilon(x)}\psi$$

$$\delta\mathcal{L} = (\partial^\mu\epsilon(x))\bar{\psi}\gamma_\mu\psi$$

gauge field $A^\mu \to A^\mu + \partial^\mu\epsilon(x)$

to obtain local invariance

We had scalar charge Q (and scalar parameter $\epsilon(x)$)

$\longrightarrow$ Spin 1 for gauge field

But now charge Q_α and

Parameter $\theta_\alpha(x)$ are spinors

Variation $\partial_\mu \theta_\alpha \sim \delta \psi_{\mu\alpha}$

has spinor and vector index

gauge particle of local SUSY has spin 3/2 and it is called gravitino

Needs spin 2 partner (and this is the graviton)

Local SUSY = Supergravity

global Lagrangian given by

$W_\alpha W^\alpha$, $S(\phi^*, V, \phi)$ and $g(\phi)$

and restricted by requirement of renormalizability

Local susy

$f_{\alpha\beta}(\phi) W^\alpha W^\beta$ $\qquad$ α, β gauge group indices (adjoint)

and Kählerpotential

$$G = 3 \log\left(-\frac{S}{3}\right) - \log(|g|^2)$$

(if one includes only terms up to two derivatives)

kinetic terms for scalar particles determined by 2^{nd} derivative of G

$$G_i{}^j = \frac{\partial G}{\partial \phi^i \partial \phi^*_j}$$

scalar potential

$$V = -\exp(-G)\left[3 + G_k (G^{-1})^k{}_l G^l\right] + \frac{1}{2} f^{-1}_{\alpha\beta} D^\alpha D^\beta$$

simplify for discussion to "minimal" kinetic terms

$$G_i{}^j = -\delta_i^j$$

$$G = -\frac{z^{*i} z_i}{M^2} - \log\frac{|g|^2}{M^6}$$

$$M = \frac{1}{\kappa} = \frac{M_{pl}}{\sqrt{8\pi}} = 2.4 \times 10^{18}\,\text{GeV}$$

$$G^i = -\frac{z^{*i}}{M^2} - \frac{g^i(z)}{g(z)}$$

$$V = \exp\left(\frac{z_i z^{*i}}{M^2}\right)\left[\left|g^i + \frac{z^{*i}}{M^2} g\right|^2 - \frac{3}{M^2}|g|^2\right] + \ldots$$

no longer positive definite

$$m_{3/2} = M \exp(-G/2) = \frac{g}{M^2} \exp\left(\frac{z_i z^{*i}}{M^2}\right)$$

for $E_{vac} = 0 \quad \longrightarrow \quad m_{3/2} = \dfrac{M_S^2}{\sqrt{3}\, M}$

possibility that $m_{3/2} \ll M_S$

local	global
$V = e^{\frac{zz^*}{M^2}}\left(\lvert f\rvert^2 - \frac{3\lvert g\rvert^2}{M^2}\right)$	$V = \lvert F\rvert^2$
$f = \frac{\partial g}{\partial z} + \frac{z^* g}{M^2}$	$F = \frac{\partial g}{\partial z}$
$M_S^2 = f \exp\left(\frac{zz^*}{M^2}\right)$	$M_S^2 = F$
$m_{3/2} \sim M_S^2 / M$	—
$\mathrm{STr}\, \mathcal{M}^2 = 2(N-1)\, m_{3/2}^2$	$\mathrm{STr}\, \mathcal{M}^2 = 0$

global limit $M^2 \to \infty$

$$\Rightarrow \quad V = |g^i|^2 = |\frac{\partial g}{\partial z_i}|^2 = FF^*$$

here auxiliary field F is replaced by $f^i = g^i + \frac{z^{*i}}{M^2} g$

and supergravity is broken if $\langle f^i \rangle \neq 0$

$$M_S^2 = \langle f \rangle \exp\left(\frac{zz^*}{M^2}\right)$$

We can have broken supergravity with vanishing cosmological constant if

$$\sum_i f^i f_i^* = \frac{3}{M^2} |g|^2 \quad \text{at minimum}$$

<u>Super-Higgs-Effect</u>: Gravitino "eats" goldstino and becomes massive

Supergravity breakdown

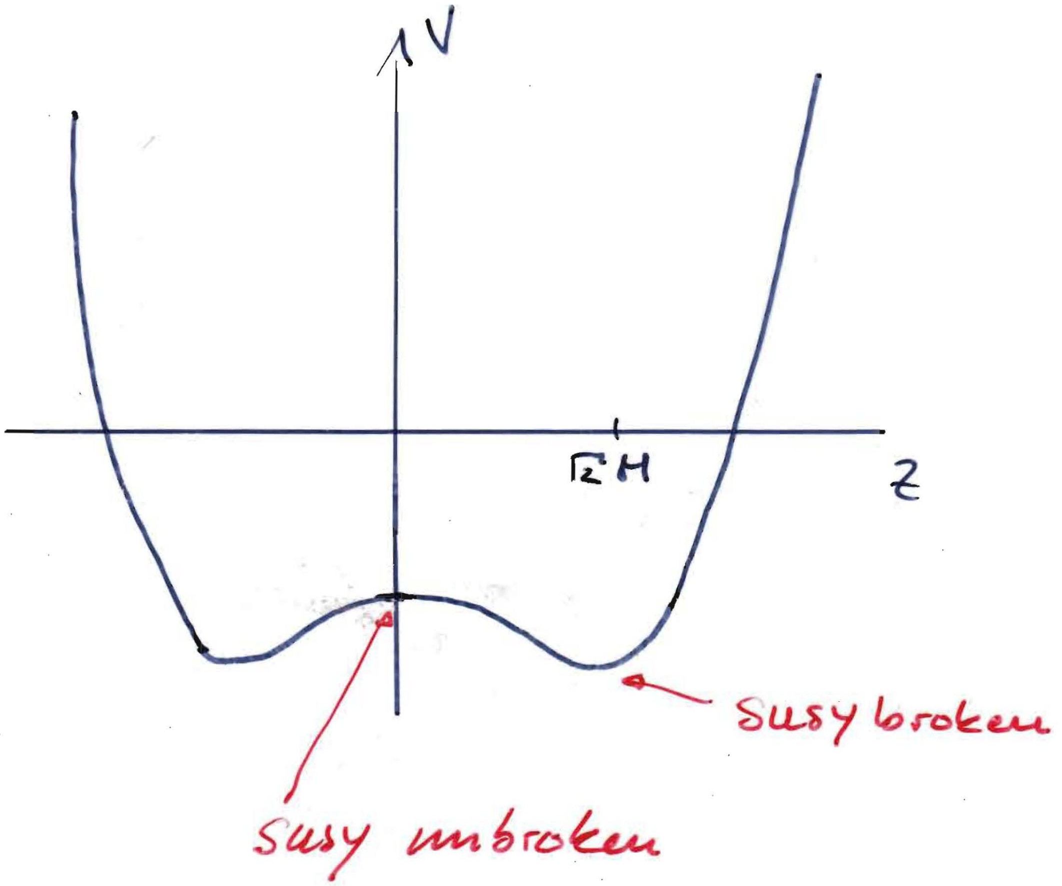
$$V = \exp\left(\frac{zz^*}{M^2}\right)\left[\left|\frac{\partial g}{\partial z} + \frac{z^*}{M^2} g\right|^2 - \frac{3}{M^2}|g|^2\right]$$

warm up example $\quad g = m^3 = \text{const}$

$$V = m^6 \exp\left(\frac{zz^*}{M^2}\right)\left[\frac{|z|^2}{M^4} - \frac{3}{M^2}\right]$$

stationary points at

$z = 0$ and $|z| = \sqrt{2}\,M$



$g = m^2(z+\beta)$

$f = m^2\left(1 + \frac{z^*(z+\beta)}{M^2}\right) \neq 0$ would signal SUSY breakdown

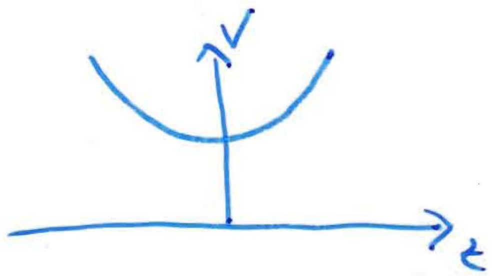
$M^2 + z^*z + z^*\beta = 0$

$z = -\frac{\beta}{2} \pm \frac{1}{2}\sqrt{\beta^2 - 4M^2}$

no solutions for $|\beta| < 2M$ and SUSY broken in this case

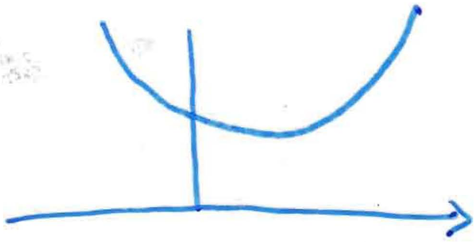
"fine tune" vacuum energy

$\beta = 0 \qquad V \sim (M^2 + |z|^2)^2 - 3M^2|z|^2 \geq 0$

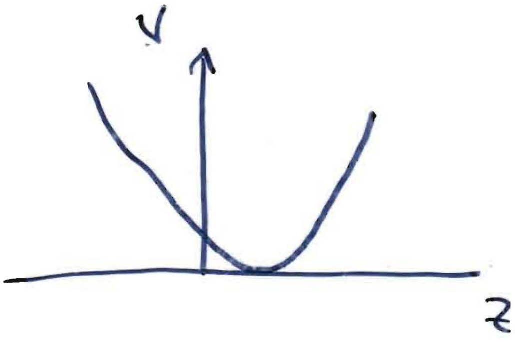


$\beta = 0$

increase β



$\beta = (2-\sqrt{3})M$



$\langle z \rangle = (\sqrt{3}-1)M$

gravitino mass $\quad m_{3/2} = \dfrac{m^2}{M} \exp\left[\dfrac{(\sqrt{3}-1)^2}{2}\right]$

scalar masses $\quad m_1^2 = 2\sqrt{3}\, m_{3/2}^2$

$m_2^2 = 2(2-\sqrt{3})\, m_{3/2}^2$

SUSY broken and $E_{vac} = 0$

(not possible in global supersymmetry)

but we still have to fine tune E_{vac}

problem of cosmological const not yet solved gaugino condensation

auxiliary field in general

$$F_i = \exp(-G/2)(G^{-1})_i^{\ j} G_j +$$
$$+ \frac{1}{4} f_{\alpha\beta k} (G^{-1})_i^{\ k} (\lambda^\alpha \lambda^\beta) + \ldots\ldots$$

$\langle\lambda\lambda\rangle \neq 0$ might break susy if

$f_{\alpha\beta}$ is nontrivial $\quad f_{\alpha\beta k} = \dfrac{\partial f_{\alpha\beta}}{\partial z^k}$

$$\Rightarrow \quad M_S^2 \sim \frac{\langle\lambda\lambda\rangle}{M}$$

and $m_{3/2} \sim \dfrac{\langle\lambda\lambda\rangle}{M^2}$ even further

suppressed

$\langle f_{\alpha\beta}\rangle \sim \dfrac{1}{g^2}$ value of g is dynamical

no breakdown in global limit

$\rightarrow$ application for strings

flat potentials

consider $G = 3 \log(\phi + \phi^*) - \log |g|^2$

and take $g = \text{const}$

$$G_i = \frac{3}{(\phi + \phi^*)} - \frac{g_i^*}{g^*}$$

$$G_i{}^j = -\frac{3}{(\phi + \phi^*)^2}$$

$$V = -e^{-G}\left[3 + G_k (G^{-1})^k_l G^l\right]$$

$$\Rightarrow \quad V \equiv 0$$

vanishes identically

but $e^{-G} = \dfrac{|g|^2}{(\phi + \phi^*)^3} \neq 0$

and SUSY is broken

$\rightarrow$ application to strings