Unification of Flavor, CP and Modular Symmetries

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Flavor symmetries

Flavor symmetries are important ingredients of the SM

- Yukawa interaction for various families
- masses and mixings for quarks and leptons
- the question of CP-symmetry and its violation

Complicated structure not well understood

- different structures in quark and lepton sectors
- CP-violation needs complexity
- Flavor symmetries are highly non-universal

Question about the origin of flavor and CP

Flavor from String Theory

String theory provides a variety of (discrete) flavor symmetries. This comes from the

- geometrical structure of extra dimensions
- string selection rules

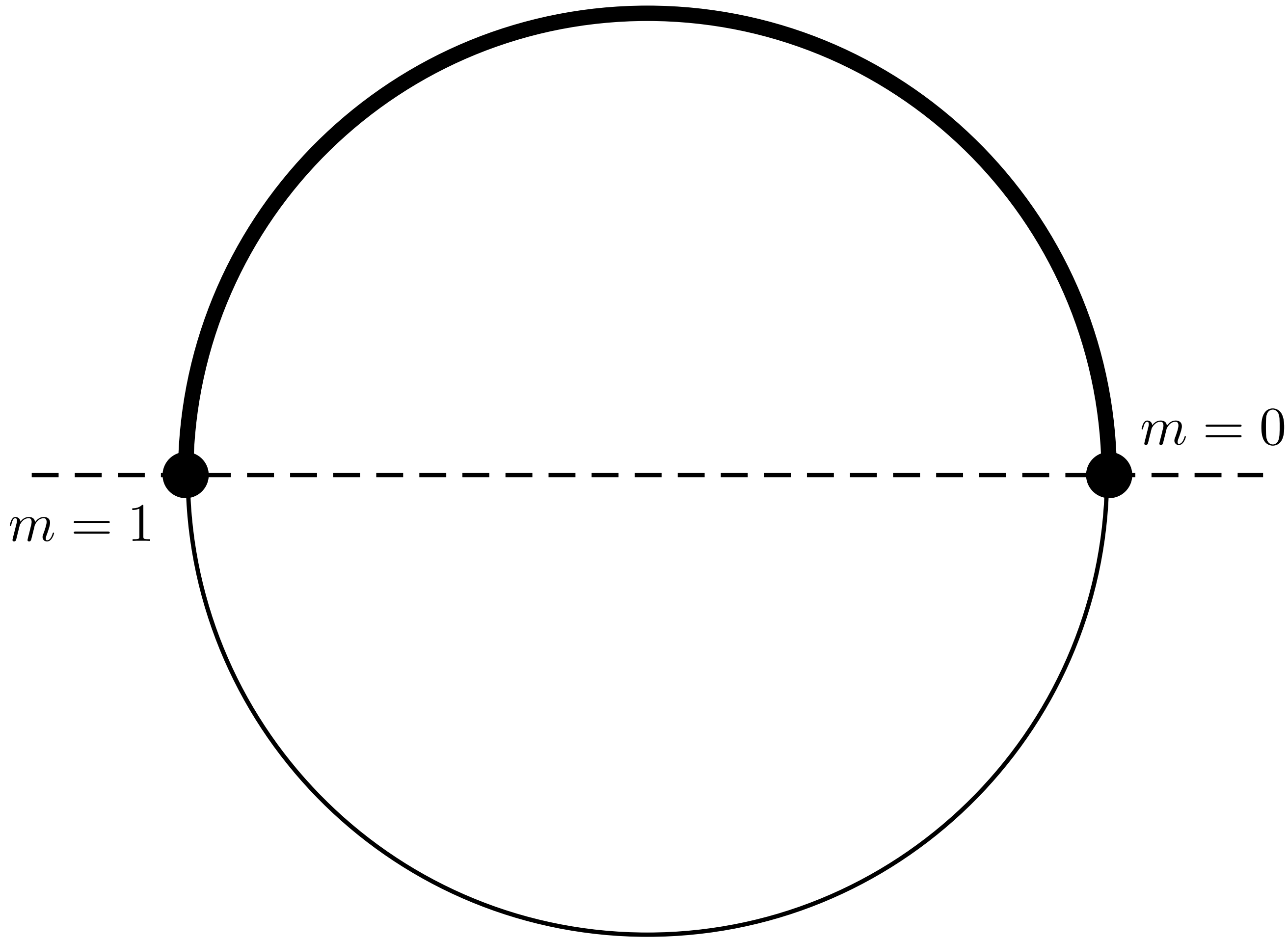
We present a new and general method to determine the flavor symmetries of string theory

- it is based on outer automorphism of the Narain space group
- it unifies flavor and CP symmetries
- it includes modular symmetries in a nontrivial way

Outline

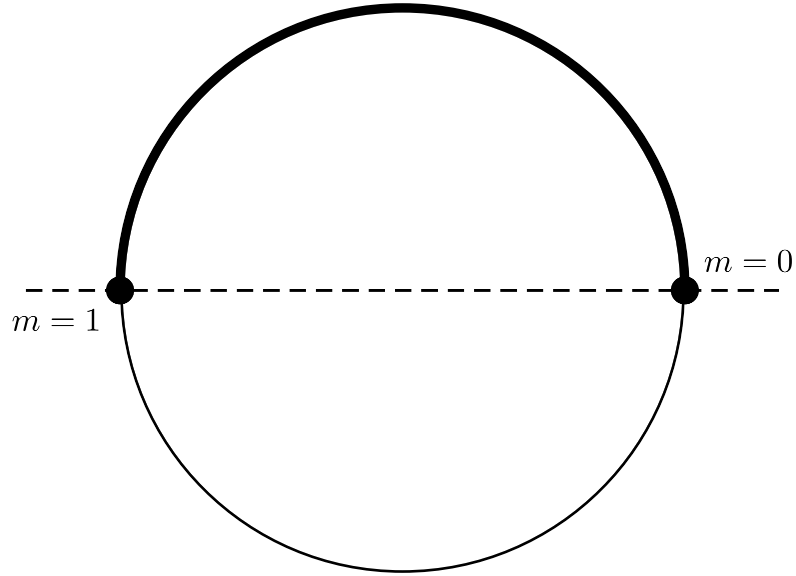
- the traditional approach to flavor symmetries via guesswork (Kobayashi, Nilles, Ploeger, Raby, Ratz, 2006)
- connections between flavor and CP (Nilles, Ratz, Trautner, Vaudrevange, 2018)
- the Narain lattice and its outer automorphisms
- modular symmetries enhance "traditional" flavor symmetries (Baur, Nilles, Trautner, Vaudrevange, 2019)
- non-universal behaviour in moduli space
- explicit example of 2d Z_3 orbifold and its "landscape" of flavor symmetries
- lessons from string theory for model building (Baur, Nilles, Trautner, Vaudrevange, to appear)

Guessing symmetries: Interval S_1/Z_2



Discrete symmetry D_4

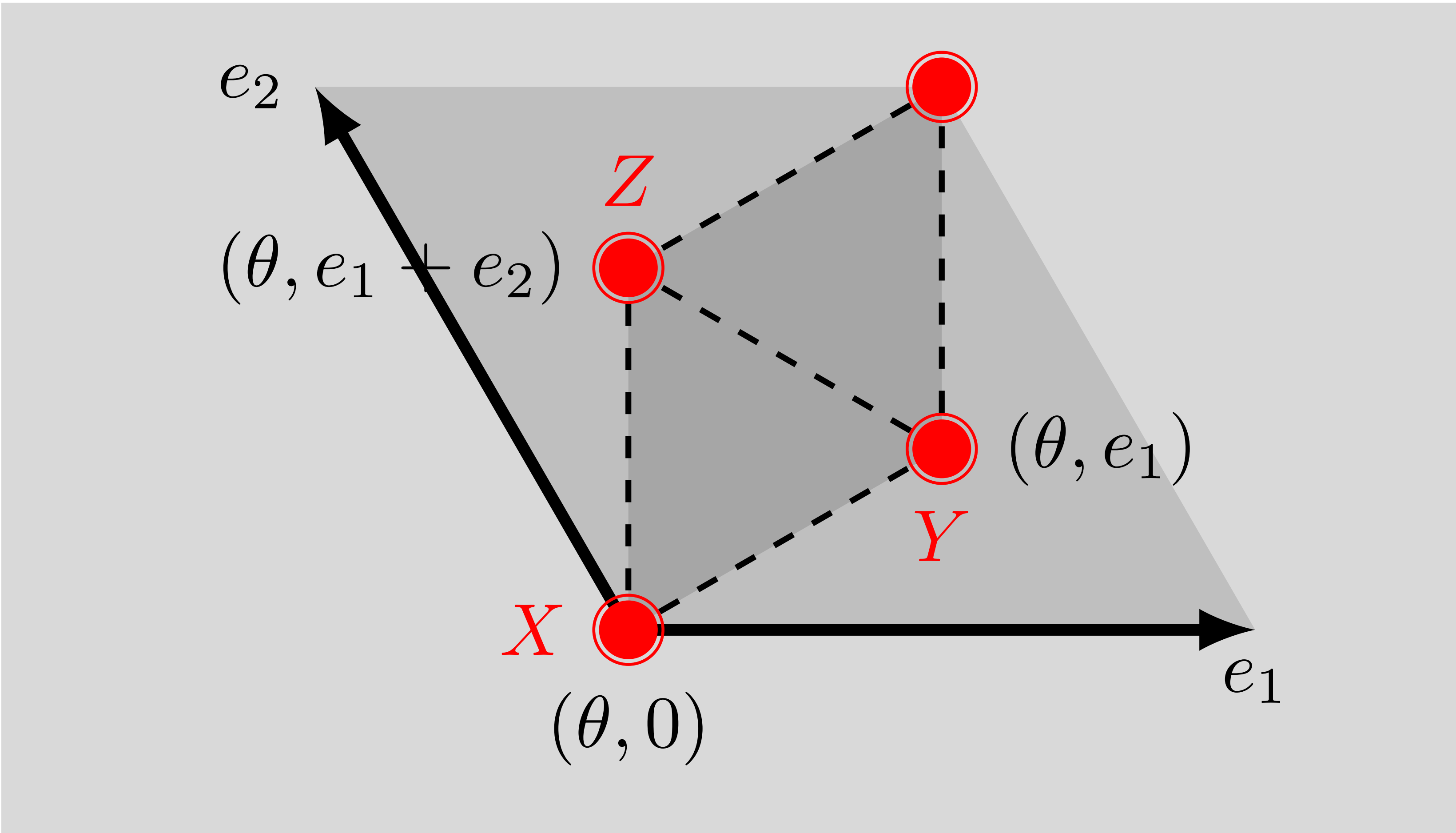
- bulk and brane fields
- S_2 symmetry from interchange of fixed points
- $Z_2 \times Z_2$ symmetry from brane field selection rules



- D_4 as multiplicative closure of S_2 and $Z_2 \times Z_2$
- D_4 – a non-abelian subgroup of $SU(2)_{\text{flavor}}$
- flavor symmetry for the two lightest families

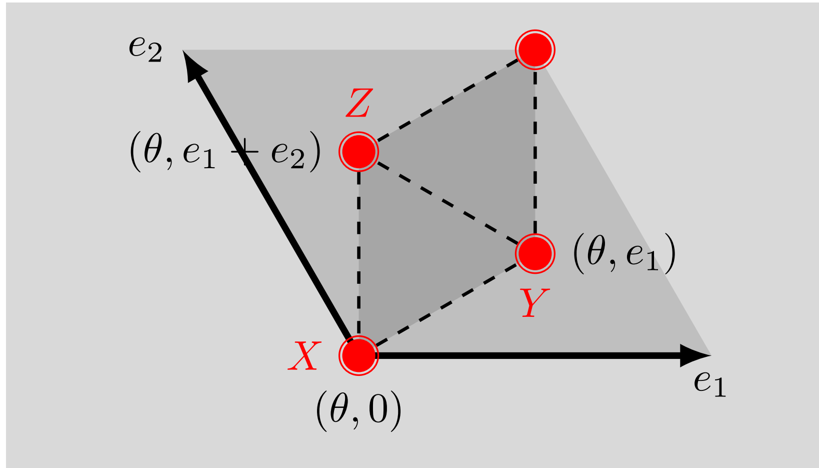
(Kobayashi, Nilles, Ploeger, Raby, Ratz, 2006)

Orbifold T_2/Z_3



Discrete symmetry $\Delta(54)$

- untwisted and twisted fields
- S_3 symmetry from interchange of fixed points
- $Z_3 \times Z_3$ symmetry from orbifold selection rules



- $\Delta(54)$ as multiplicative closure of S_3 and $Z_3 \times Z_3$
- $\Delta(54)$ – a non-abelian subgroup of $SU(3)_{\mathrm{flavor}}$
- flavor symmetry for three families of quarks and leptons

(Kobayashi, Nilles, Ploeger, Raby, Ratz, 2006)

$\Delta(54)$ group theory

$\Delta(54)$ is a non-abelian group and has representations:

- one trivial singlet 1_0 and one nontrivial singlet 1_-
- two triplets 3_1, 3_2 and corresponding anti-triplets $\bar{3}_1$, $\bar{3}_2$
- four doublets 2_k $(k = 1, 2, 3, 4)$

Some relevant tensor products are:

- $3_1 \otimes \bar{3}_1 = 1_0 \oplus 2_1 \oplus 2_2 \oplus 2_3 \oplus 2_4$
- $2_k \otimes 2_k = 1_0 \oplus 1_- \oplus 2_k$

$\Delta(54)$ is a good candidate for a flavour symmetry.

But where is CP?

CP as outer automorphism

Outer automorphisms map the group to itself but are not group elements themselves

- $\Delta(54)$ has outer automorphism group S_4
- CP could be Z_2 subgroup of this S_4
- Physical CP transforms (rep) to $(rep)^*$

This gives an intimate relation of flavour and CP symmetry

- CP broken due to the presence of winding modes
- lepto-genesis through decay of winding modes
- CP-violation à la CKM via field dependent Yukawa couplings

(Nilles, Ratz, Trautner, Vaudrevange, 2018)

Search for a general method

We have seen that even in simple systems we obtain sizeable flavor groups

- D_4 for the interval
- $\Delta(54)$ for the 2-dimensional Z_3 orbifold

Did we find the complete flavor symmetry in these cases?

In reality we have even six compact dimensions

- variety of complicated group structures possible
- dependence on localisation of fields in extra dimensions

A general mechanism to find the complete set of flavor symmetries is based on the

outer automorphisms of the Narain space group.

(Baur, Nilles, Trautner, Vaudrevange, 2019)

The Narain Lattice

In the string there are D right- and D left-moving degrees of freedom $Y = (y_R, y_L)$. Y compactified on a $2D$ torus

$$Y = \begin{pmatrix} y_R \\ y_L \end{pmatrix} \sim Y + E\hat{N} = \begin{pmatrix} y_R \\ y_L \end{pmatrix} + E \begin{pmatrix} n \\ m \end{pmatrix}$$

defines the Narain lattice with

- the string's winding and Kaluza-Klein quantum numbers n and m
- the Narain vielbein matrix E that depends on the moduli of the torus: radii, angles and anti-symmetric tensor fields B.

The Narain Space Group

A Z_K orbifold with twist Θ leads to the identification

$$Y \sim \Theta^k Y + E\hat{N} \quad \text{where} \quad \Theta = \begin{pmatrix} \theta_R & 0 \\ 0 & \theta_L \end{pmatrix} \quad \text{and} \quad \Theta^K = 1$$

with θ_L, θ_R elements of $SO(D)$. For a symmetric orbifolds $\theta_L = \theta_R$ (we do not include roto-translations here).

The Narain space group $g = (\Theta^k, E\hat{N})$ is then generated by

$$\text{twists} \quad (\Theta, 0) \quad \text{and shifts} \quad (1, E_i) \quad \text{for} \quad i = 1 \ldots 2D$$

Outer automorphisms map the group to itself but are not elements of the group.

Modular Transformations

Modular transformation exchange windings and momenta and act nontrivially on the moduli of the torus.

In $D = 2$ these transformations are connected to the group $SL(2, Z)$ acting on Kähler and complex structure moduli.

The group $SL(2, Z)$ is generated by two elements

$$S \; , \; T : \quad \text{with} \;\; S^4 = 1 \;\; \text{and} \;\; S^2 = (ST)^3$$

On a modulus M with have the transformations

$$S : \quad M \to -\frac{1}{M} \quad \text{and} \quad T : M \to M + 1$$

Further transformations might include $M \to -\overline{M}$ and mirror symmetry between Kähler and complex structure moduli.

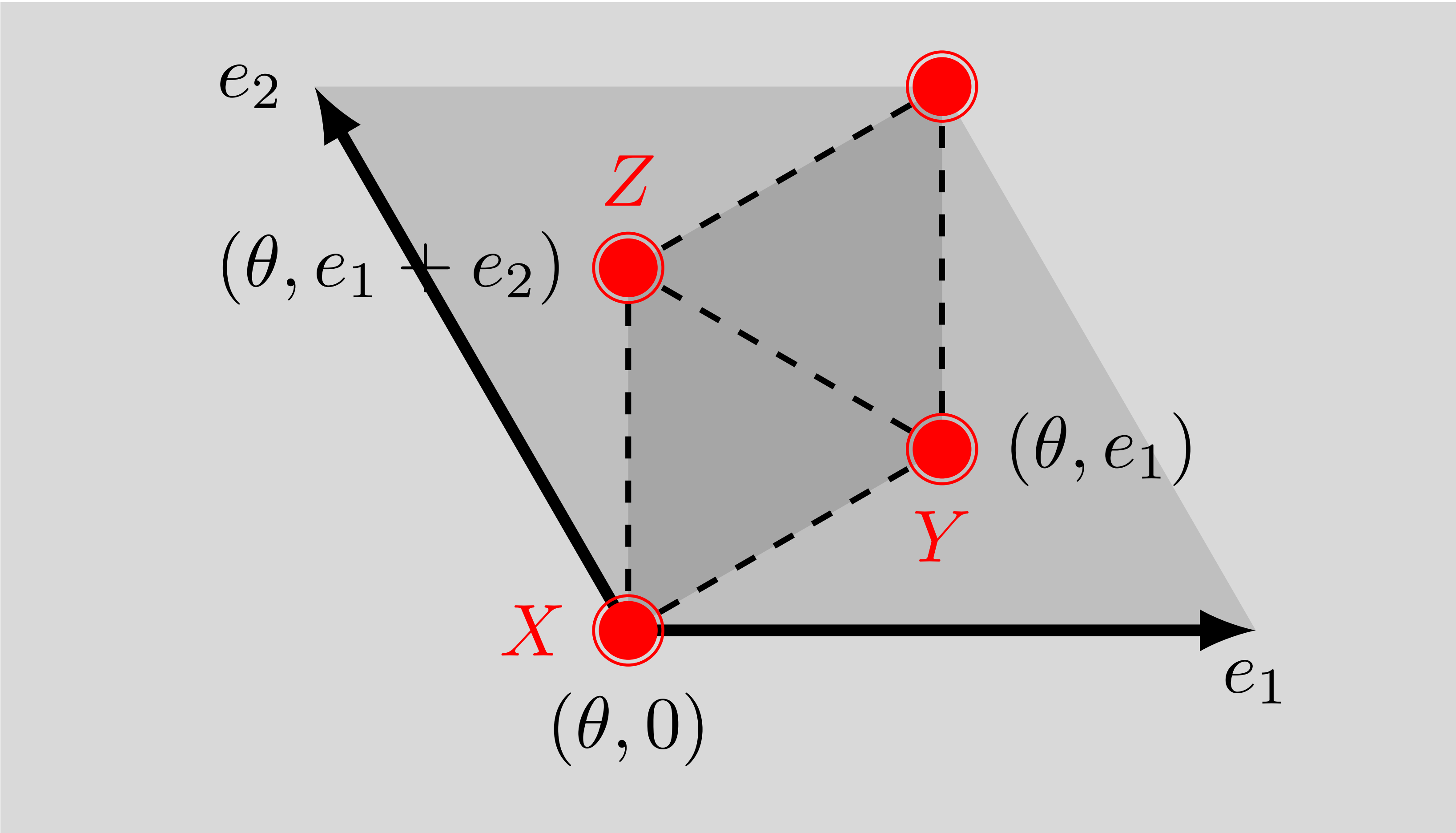
Candidate symmetries

As outer automorphisms of the Narain space group we might find

- traditional flavor symmetries which are universal in moduli space
- a subset of the modular transformations that act as symmetries at specific "points" in moduli space
- at these "points" we shall have an enhanced symmetry that combines the traditional flavor symmetry with some of the modular symmetries

The full flavor symmetry is non-universal in moduli space
At generic points in moduli space we have the universal traditional flavor symmetry

Orbifold T_2/Z_3



Example: T_2/Z_3 Orbifold

On the orbifold some of the moduli are frozen

- lattice vectors e_1 and e_2 have the same length
- angle is 120 degrees

Modular transformations form a subgroup of $SL(2,Z)$

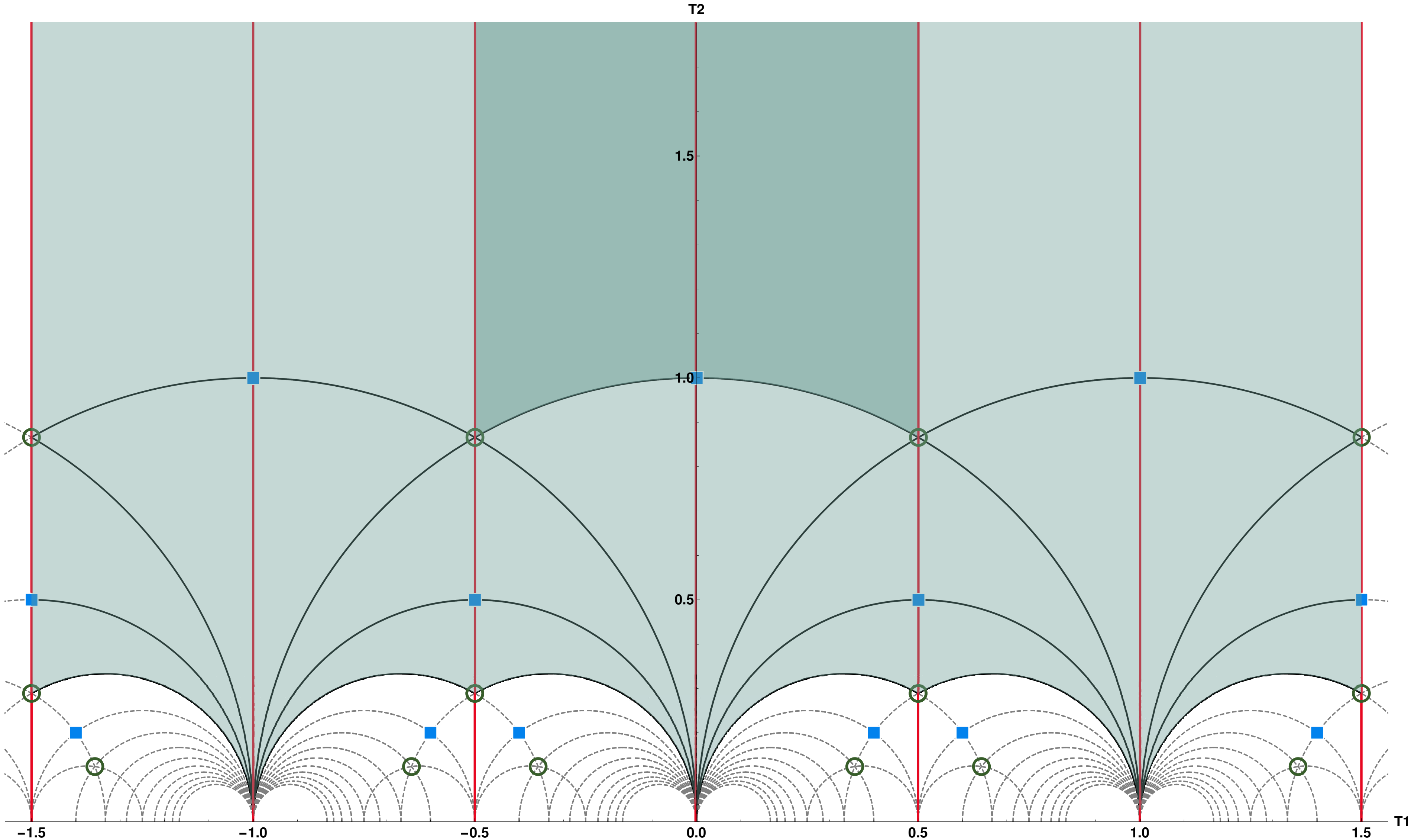
- $\Gamma(3)$ as a mod(3) subgroup of $SL(2,Z)$; ($\Gamma(3) = A_4$)
- $\Gamma(3)$ acts on the moduli
- twisted fields transform under a bigger group T', (similar to enhancement of $SO(3)$ to $SU(2)$ for spinors)

(Lauer, Mas, Nilles, 1989; Lerche, Lüst, Warner, 1989)

- transformation $M \to -\overline{M}$ completes the picture

Full group is SG(48,29) with 48 elements

Moduli space of $\Gamma(3)$



Flavour Symmetries I

Generic point in moduli space.

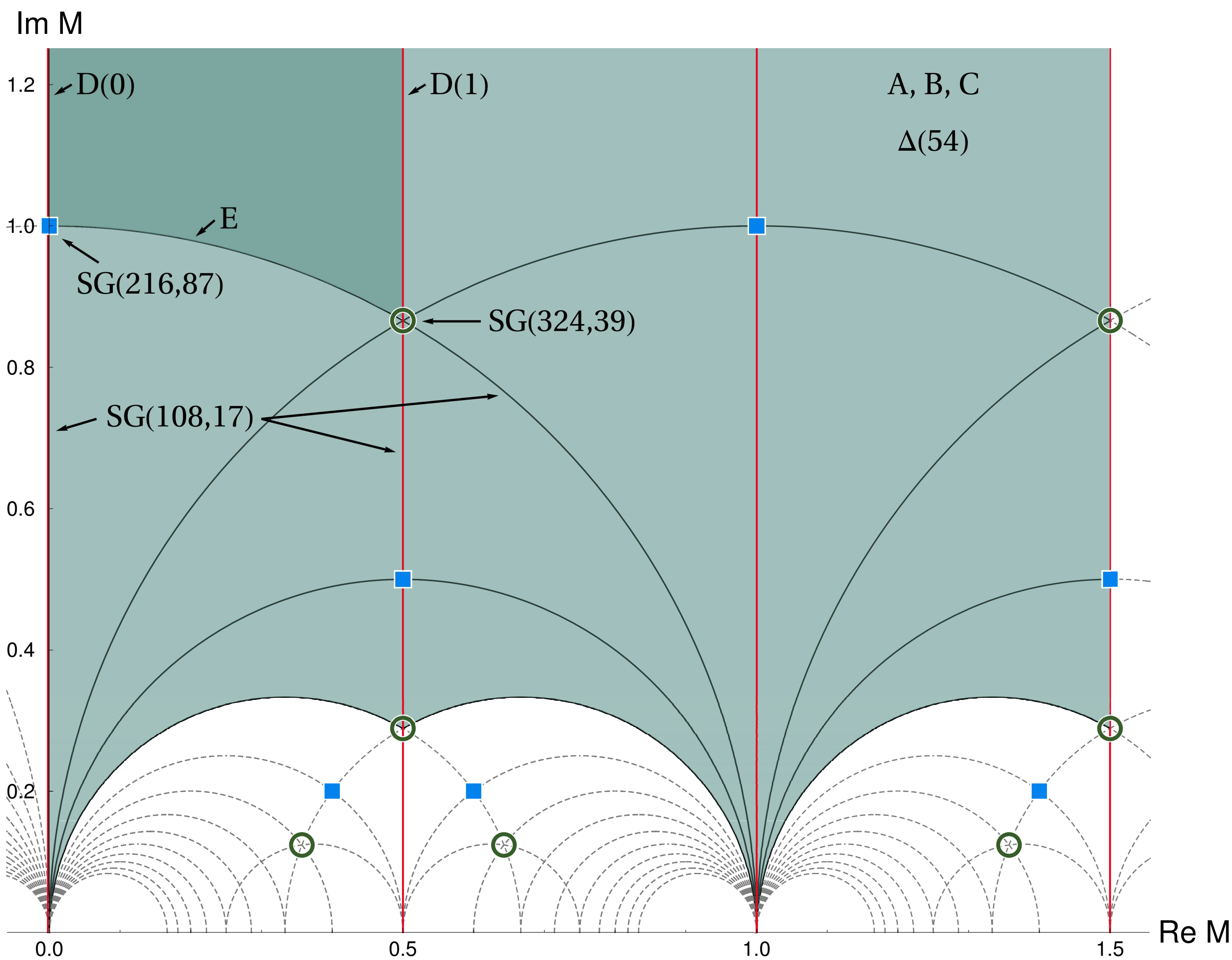
Outer automorphisms of the Narain space group are

- shift $A = (1_4; \frac{1}{3}, \frac{2}{3}, 0, 0)$
- and shift $B = (1_4; 0, 0, \frac{1}{3}, \frac{1}{3})$
- a left-right symmetric rotation $C = (-1_4; 0, 0, 0, 0)$

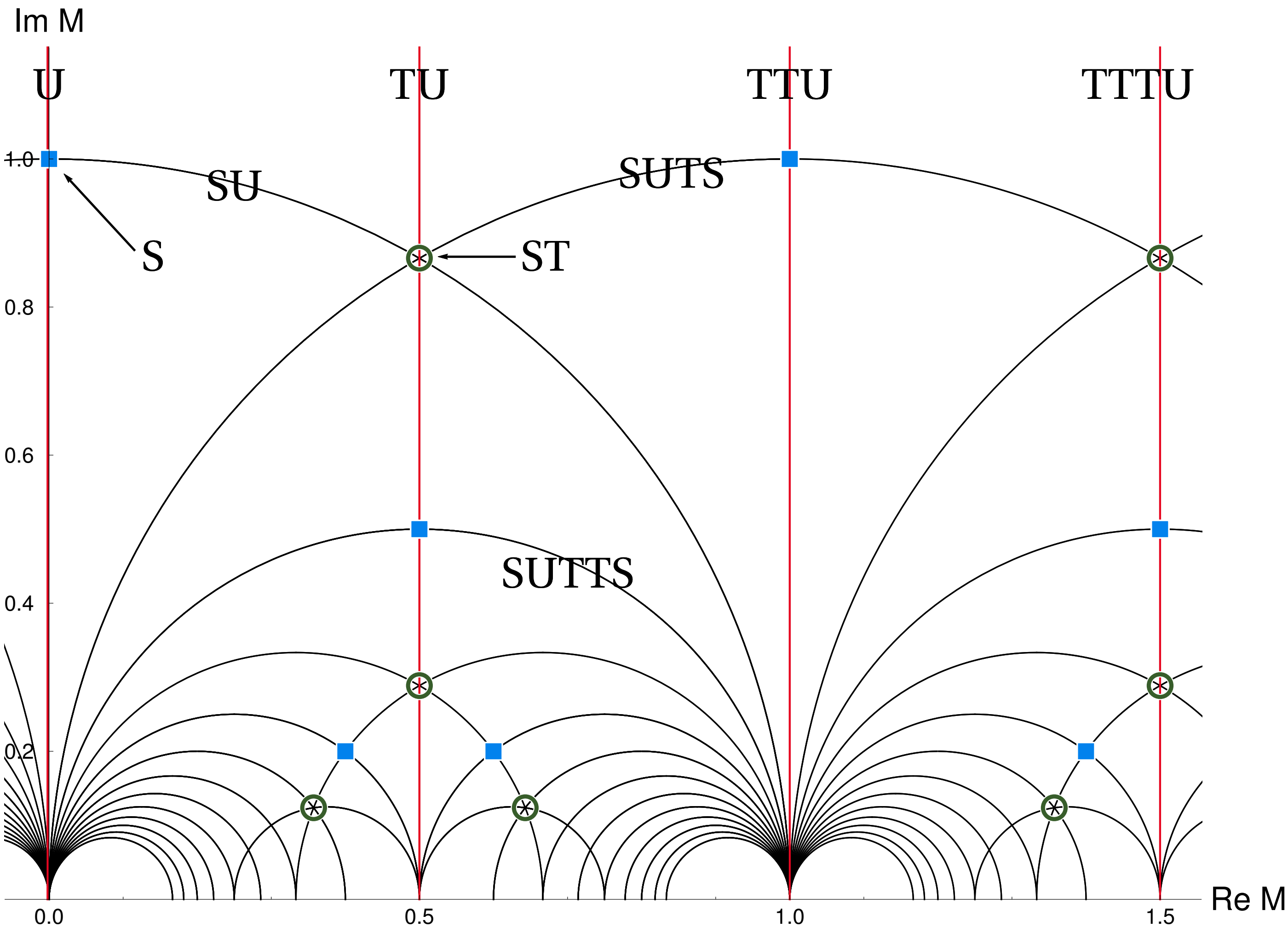
Multiplicative closure of A, B and C leads to $\Delta(54)$.

- the earlier guesswork gave the correct result!
- but the new method produces the result automatically
- can be generalised easily to more complicated situations (like, e.g. six dimensions)

Moduli space of flavour groups



Fixed lines and points



$$S : M \to -\frac{1}{M}, \quad T : M \to M + 1 \quad \text{and} \quad U : M \to -\overline{M}$$

Flavour Symmetries II

The red lines:

These are fixed lines under T and U. We have

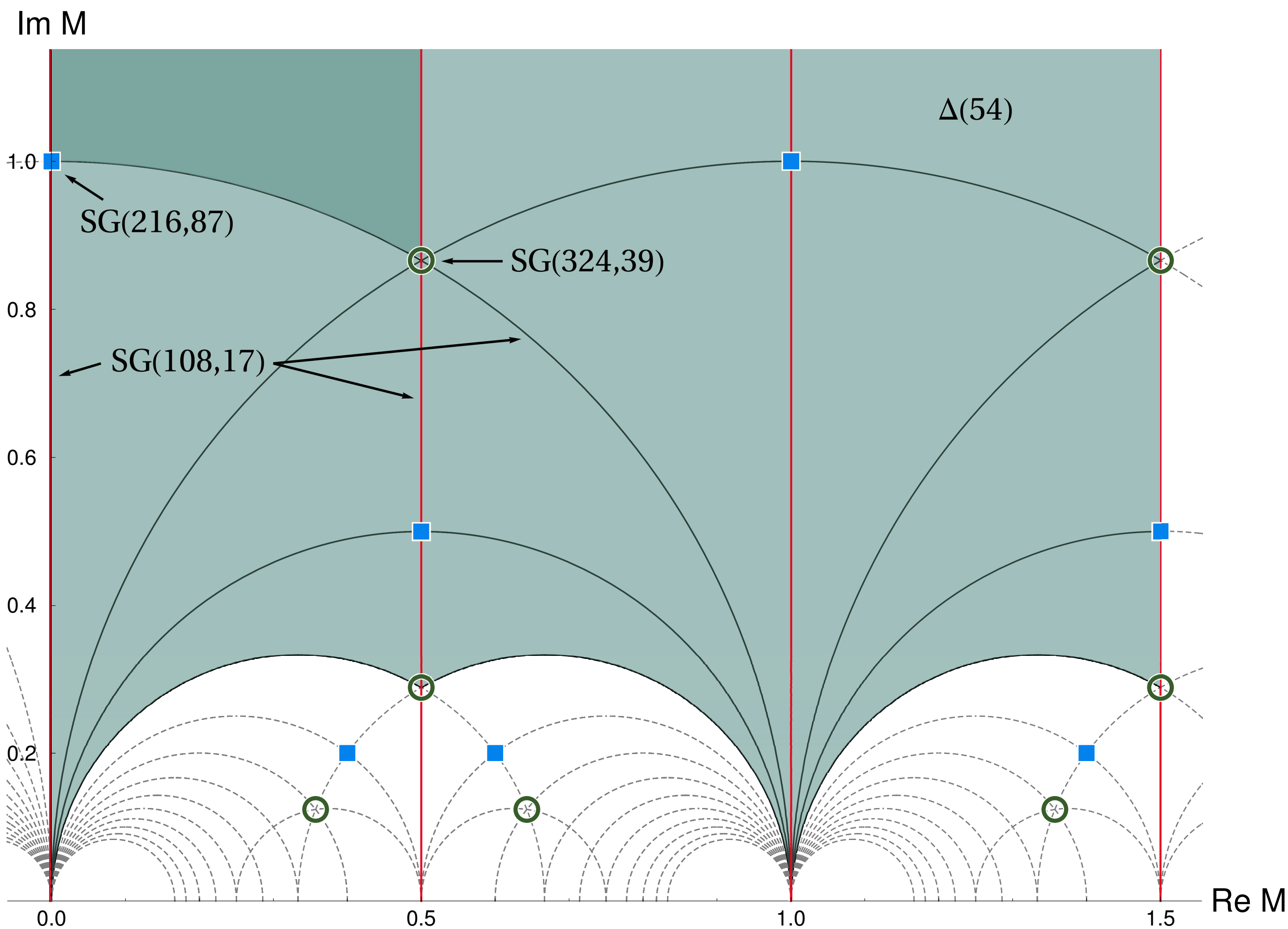
- again A, B, and C
- and a left-right symmetric reflection D

Multiplicative closure of leads to $SG(108, 17)$. This includes the formerly discussed CP-transformation! Unification of flavor and CP (spontaneous breakdown away from the line).

The circles: e.g. fixed lines under S and U

- new asymmetric reflection E (instead of D)
- again $SG(108, 17)$ but differently aligned
- enhanced with different Z_2 from $S_4 = \mathrm{Out}(\Delta(54))$

Moduli space of flavour groups



Flavour Symmetries III

Blue squares: two lines meet

- enhancement to $SG(216, 87)$

The small circles: three lines meet

- maximum enhancement to $SG(324, 39)$

The modular group T' has 24 elements, but not all of them lead to an enhancement of the flavor group $\Delta(54)$.

Only the elements within S_4 of the outer automorphisms of $\Delta(54)$ are relevant

- this leads to unification of flavour and CP
- CP exact at those fixed lines and points

Messages

We have designed a generic method to find all flavor symmetries (based on the Narain space group)

- unification of traditional (discrete) flavor, CP und modular symmetries
- traditional flavor symmetry is universal in moduli space
- there are non-universal enhancements (including CP at some places (broken in generic moduli space))
- not the full modular transformations can appear as symmetries
- the potential flavor groups are large (in our case already up to $SG(324,39)$ for two extra dimensions)

Consequences

This opens a new arena for flavor model building

- a new look at CP as discrete gauge symmetry

  (Nilles, Ratz, Trautner, Vaudrevange, 2018)

- modular symmetries for flavor (Altarelli, Feruglio, 2006; Feruglio, 2017)
- groups are large and allow for flexibility (Hagedorn, König, 2018)
- the concept of local flavor symmetries allows different flavor groups for different sectors of the theory

  (Baur, Nilles, Trautner, Vaudrevange, 2019)

- non-universal structure from modular symmetries (there is still the traditional universal flavor group)
- different flavor symmetries for quarks and leptons are no surprise