Origin of CP and Flavor

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Do we need CP as a Symmetry?

CP (and its violation) is relevant for several phenomena:

- CP violation in standard model (CKM phase)
- matter-antimatter asymmetry of the universe
- the strong CP-problem ($\Theta_{\rm QCD}$)

The strong CP-problem requires CP to be a symmetry

- Origin of CP symmetry? How is it broken?
- Is it related to flavour symmetries?

CP: you have to make it and to break it.

Is there a top-down explanation?

Outline I

We want to consider CP as a discrete gauge symmetry.
To motivate this we shall discuss

- the nature of global versus local symmetries
- the fate of global symmetries
- global symmetries in Standard Model of particle physics
- CP as a discrete gauge symmetry
- discrete CP in string theory (as UV-completion)
- an example for CP symmetry and its breakdown
- Lepto-genesis through decay of heavy particles
- Solution of the strong CP-problem

(Nilles, Ratz, Trautner, Vaudrevange, 2018)

Outline II

We want to understand flavor and CP symmetries from string theory

- develop a general mechanism to deduce flavor symmetries from string models
- the Narain lattice and Narain space group
- outer automorphism of the Narain space group
- duality (modular) transformations enter the game
- enhanced flavor symmetries
- CP as a modular symmetry
- flavor is non-universal in moduli space

(Baur, Nilles, Trautner, Vaudrevange, 2019)

Global versus Gauge Symmetries

Both types of symmetries are observed in nature

- Global symmetries as a symmetry of physical system
- Local (gauge) symmetry as a "redundancy of description"
- role of anomalies differs in both cases

Global and local symmetries appear in the Standard model of particle physics:

- are gauge symmetries more fundamental?
- are global symmetries necessarily approximate?

Is there a concept of discrete gauge symmetries?

Golden Age of global symmetries

Once upon a time gobal symmetries were "sacred", e.g.

- Baryon an Lepton number
- discrete symmetries like C, P and T
- we here assume CPT to be exact (thus CP$\sim$T)

It was a kind of "shock" when

- P and C violation was found in the 1950'ties
- and CP violation in the early 1960'ties

Why does nature spoil these beautiful symmetries?

Change of perspective with the rise of the Standard Model!

Standard Model of Particle Physics

In the standard model gauge theories (instead of global symmetries) are of primary importance

- $SU(3)_{QCD} \times SU(2)_W \times U(1)_Y$ as gauge symmetries of strong, weak and electromagnetic interactions
- P and C are "maximally" violated as a consequence of the spectrum of the theory (Weyl-fermions)
- CP and T are broken by Yukawa-couplings of three families of quarks and leptons

Remnant global symmetries as e. g. related to Baryon and Lepton number appear as low energy accidents and are thus an indirect consequence of the gauge symmetries.

Baryon and Lepton number

The global symmetries $U(1)_B$ and $U(1)_L$

- are good symmetries at the renormalizable level, i.e. operators of dimension four (or less than four)

but broken by higher dimensional operators

- $\frac{1}{M}HHLL$ of dimension 5
- neutrino mass as $\frac{<H^2>}{M}$ via "see saw" mechanism
- $\frac{1}{M^2}QQQL$ of dimension 6 (relevant for proton decay)

$U(1)_{B,L}$ are accidental low energy consequences of the gauge symmetries of the standard model.

Role of C, P and T

The relation of C, P and T to gauge theories is more subtle:

- conserved in electromagnetic interactions
- conserved in perturbative QCD but broken by non-perturbative effects related to $\Theta_{\rm QCD}$
- P and C incompatible with spectrum of $SU(2)_W$
- CP violation in Yukawa couplings for at least 3 families of quarks and leptons (some complexity needed)

"Non-perturbative" topological term in QCD violates T

$$\Theta \epsilon^{\mu\nu\rho\sigma} {\rm Tr} F_{\mu\nu} F_{\rho\sigma}$$

leads to electric dipole moment of neutron. This is known as the "strong CP problem" (possible solution via axion)

Paradigm shift from global to local

Today gauge symmetries are of primary importance. Global symmetries are most probably low-energy "accidents" and not exact symmetries.

- $U(1)_{B,L}$ in the standard model are anomalous and broken by non-perturbative effects
- gauge symmetries are anomaly free
- the inflationary universe dilutes all matter. We need B,L-violation (as well as CP-violation) to create a baryon /lepton asymmetry
- black hole evaporation is conjectured to violate all global symmetries (while charges of gauge symmetries are conserved due to presence of gauge-flux).

Characteristics of gauge theories

What is so special about gauge symmetries?

- non-perturbative topological objects like monopoles, strings and branes
- duality symmetries like electric-magnetic duality E.M.
- Aharonov-Bohm effect and flux integrals at infinity

Constraints from "quantum gravity"

- the role of gauge theories in AdS/CFT correspondence
- spectrum completeness conjecture
- the weak gravity conjecture and the "swampland"

All of these requirements seem to be fulfilled in string theory

The concept of discrete gauge theories

Is there a distinction between global and local discrete symmetries?

- subgroup of continuous gauge symmetry
- subject to anomaly cancellation as well

Concept of discrete gauge symmetry is more general

- presence of non-perturbative topological objects
- presence of discrete flux integrals at infinity
- discrete symmetries from higher order gauge fields

Again, string theory can be used as a tool to study these generalized discrete gauge symmetries

CP as a discrete gauge symmetry

CP (violation) is relevant for several phenomena:

- the strong CP-problem (Θ_{QCD})
- CP violation in standard model (CKM phase)
- matter-antimatter asymmetry of the universe

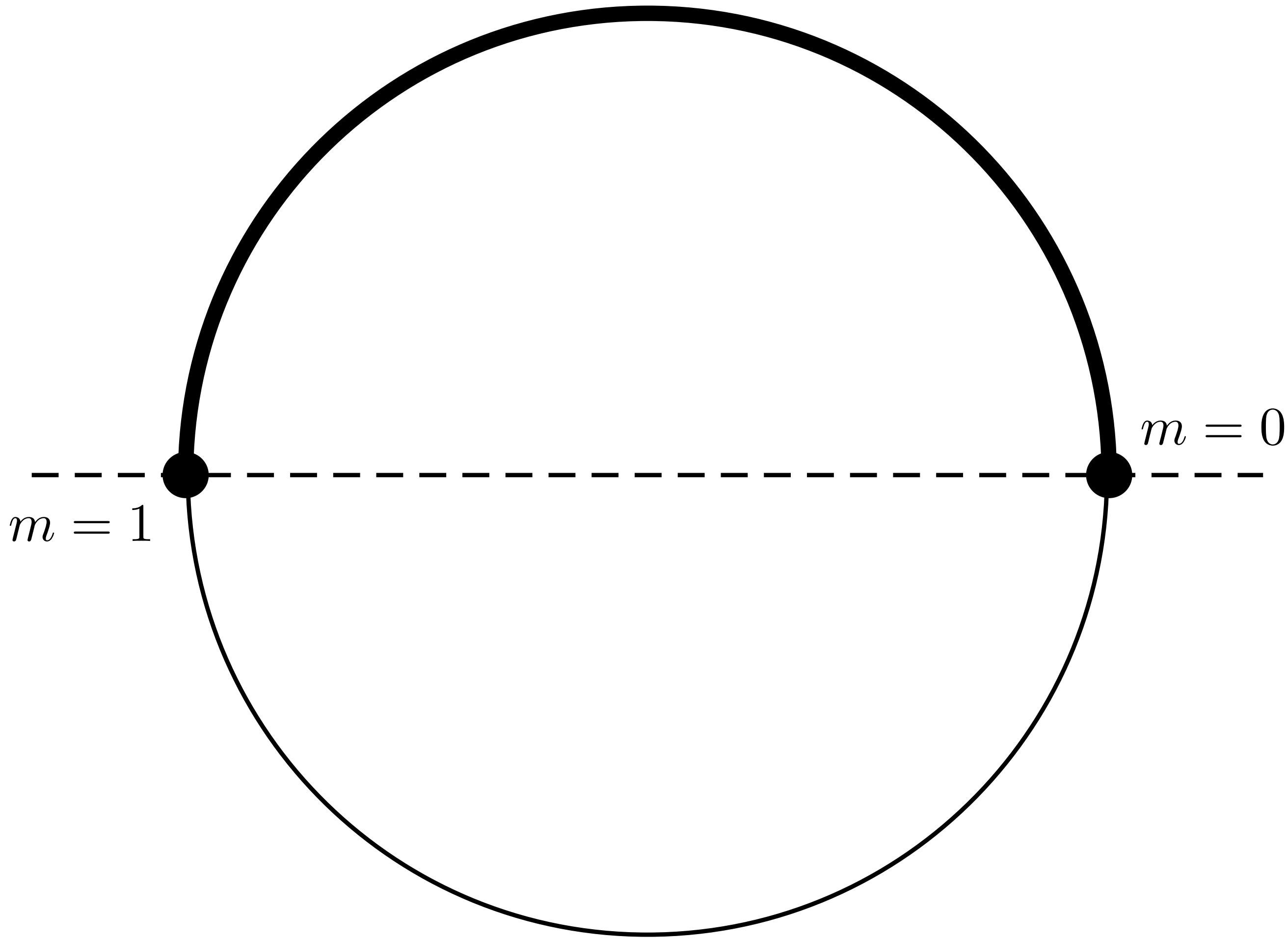
The strong CP-problem requires CP to be a symmetry

- Origin of CP symmetry? How is it broken?
- Is it related to flavour symmetries?

CP: make it and break it. Is there a top-down explanation?

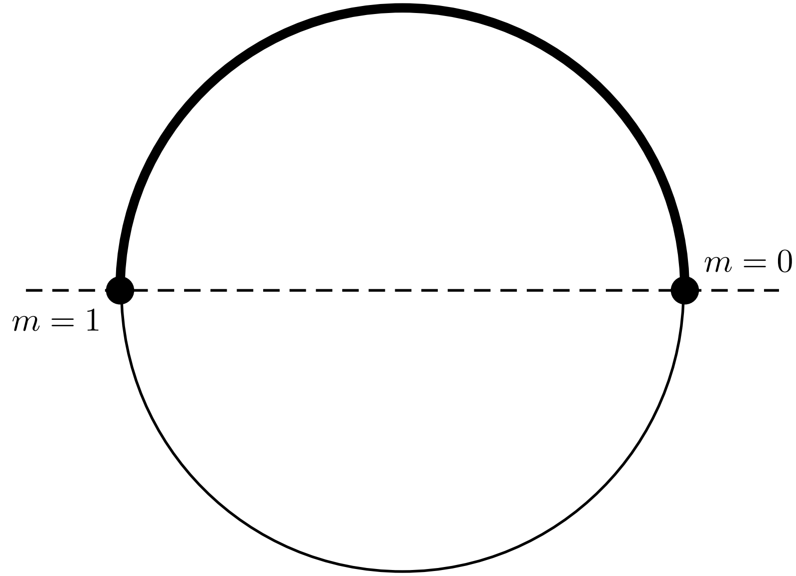
We use string theory for a consistent UV-completion.

Interval S_1/Z_2



Discrete symmetry D_4

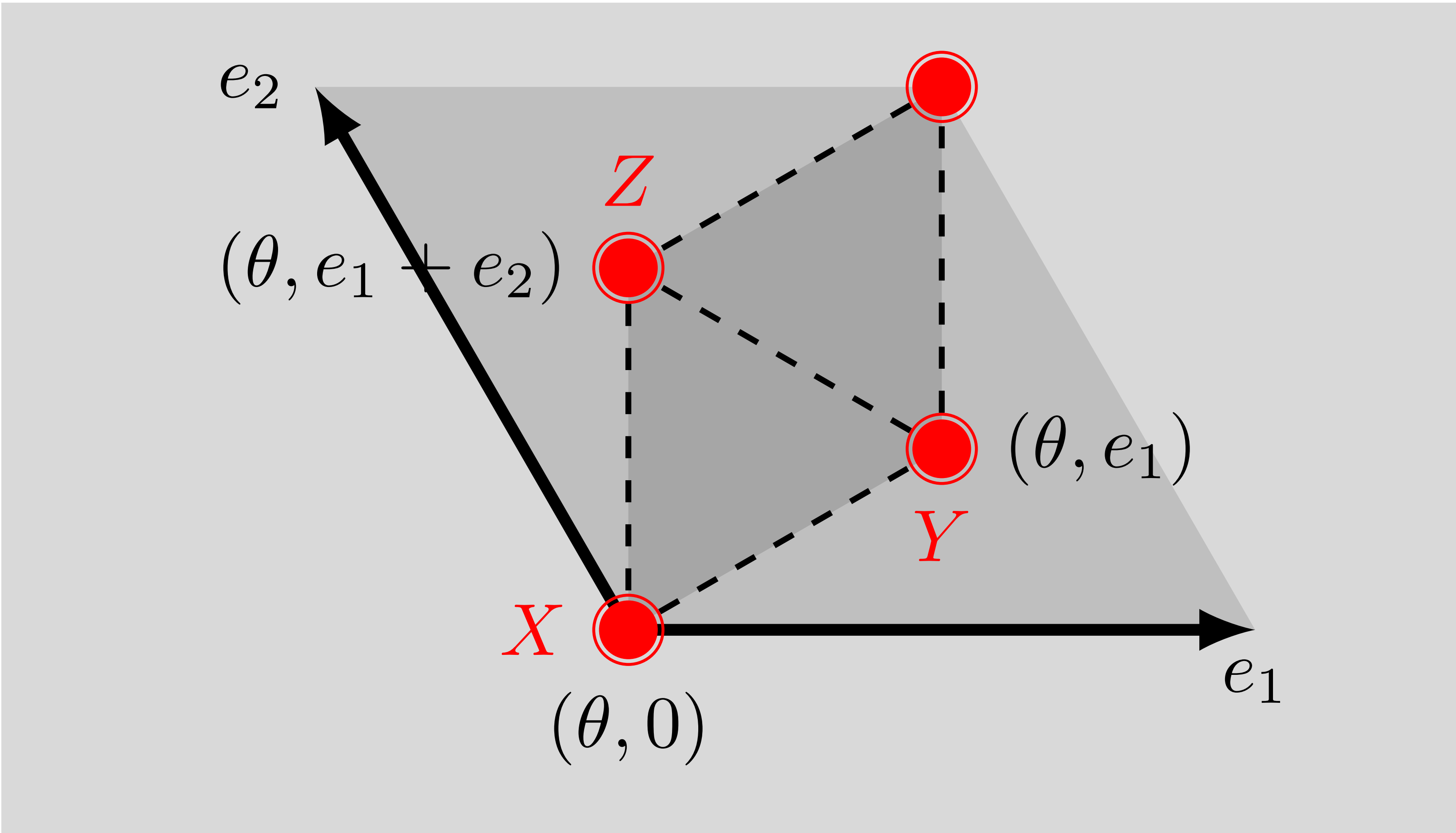
- bulk and brane fields
- S_2 symmetry from interchange of fixed points
- $Z_2 \times Z_2$ symmetry from brane field selection rules



- D_4 as multiplicative closure of S_2 and $Z_2 \times Z_2$
- D_4 – a non-abelian subgroup of $SU(2)_{\text{flavor}}$
- flavor symmetry for the two lightest families

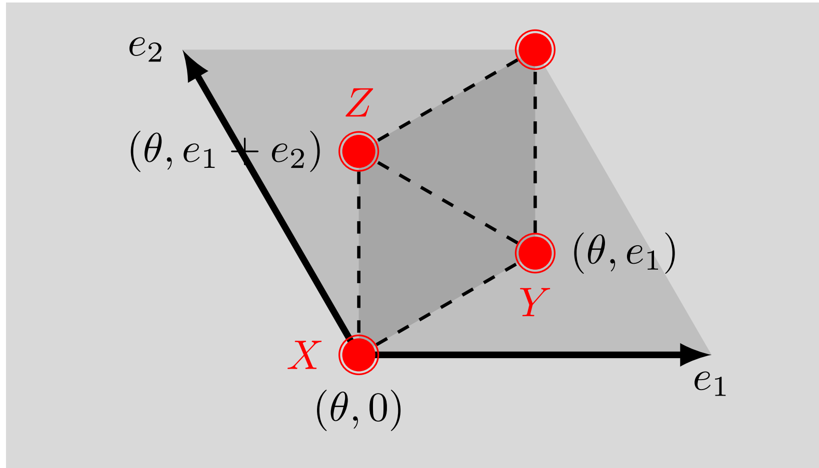
(Kobayashi, Nilles, Ploeger, Raby, Ratz, 2006)

Orbifold T_2/Z_3



Discrete symmetry $\Delta(54)$

- untwisted and twisted fields
- S_3 symmetry from interchange of fixed points
- $Z_3 \times Z_3$ symmetry from orbifold selection rules



- $\Delta(54)$ as multiplicative closure of S_3 and $Z_3 \times Z_3$
- $\Delta(54)$ – a non-abelian subgroup of $SU(3)_{\text{flavor}}$
- flavor symmetry for three families of quarks and leptons

(Kobayashi, Nilles, Ploeger, Raby, Ratz, 2006)

$\Delta(54)$ group theory

$\Delta(54)$ is a non-abelian group and has representations:

- one trivial singlet 1_0 and one nontrivial singlet 1_-
- two triplets 3_1, 3_2 and corresponding anti-triplets $\bar{3}_1$, $\bar{3}_2$
- four doublets 2_k $(k = 1, 2, 3, 4)$

Some relevant tensor products are:

- $3_1 \otimes \bar{3}_1 = 1_0 \oplus 2_1 \oplus 2_2 \oplus 2_3 \oplus 2_4$
- $2_k \otimes 2_k = 1_0 \oplus 1_- \oplus 2_k$

$\Delta(54)$ is a good candidate for a flavour symmetry.

But where is CP?

CP as outer automorphism

Outer automorphisms map the group to itself but are not group elements themselves

- $\Delta(54)$ has outer automorphism group S_4
- CP could be Z_2 subgroup of this S_4
- Physical CP transforms (rep) to $(rep)^*$

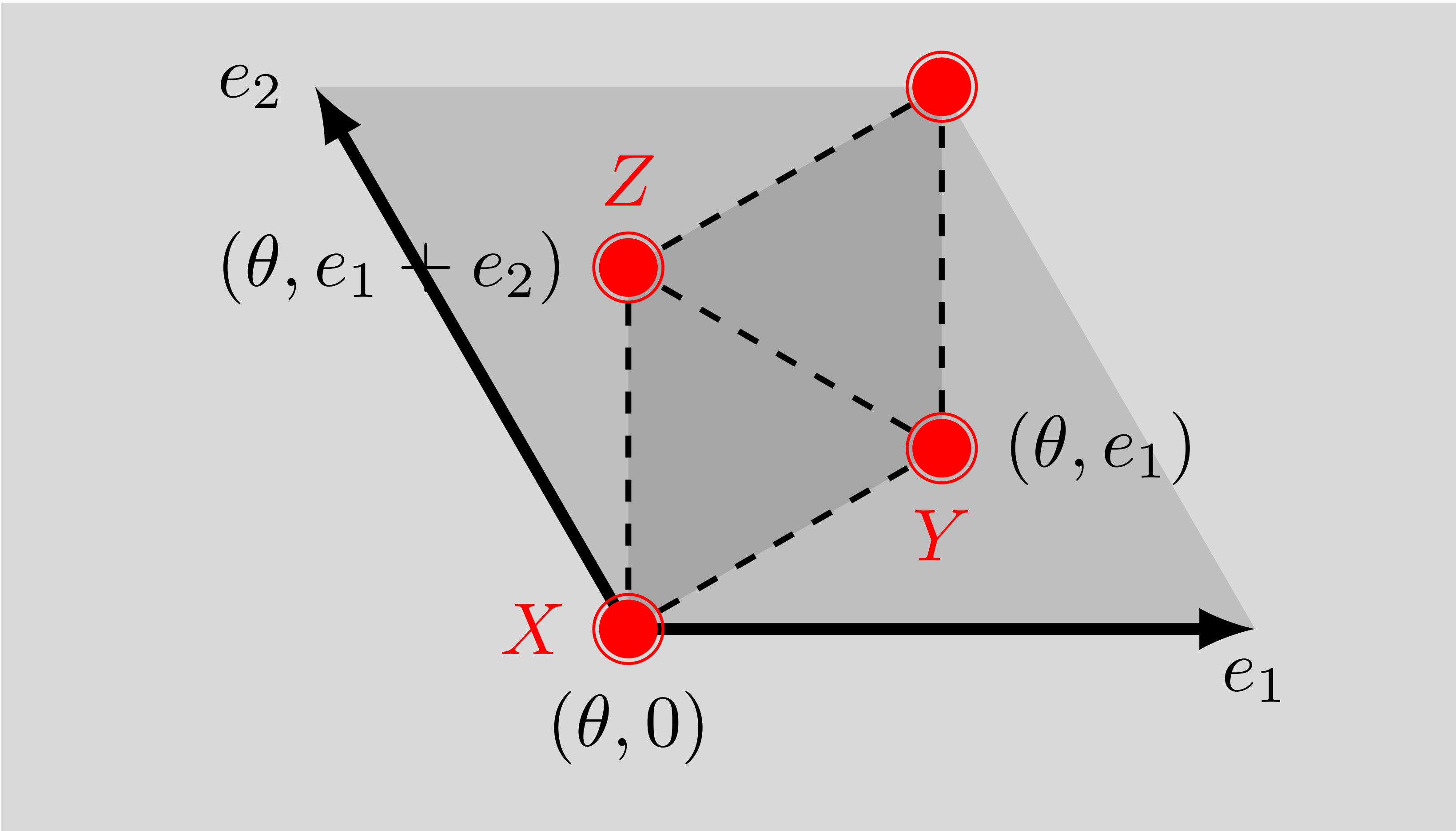
This gives an intimate relation of flavour and CP symmetry

- possible obstructions for a successful definition of CP
- controlled by "representation content" of the symmetry
- could lead to "explicit geometric CP violation"

(Holthausen, Lindner, Schmidt, 2012;

Chen, Fallbacher, Mahanthappa, Ratz, Trautner, 2014)

Orbifold T_2/Z_3



T_2/Z_3 orbifold examples

We label a string state by its constructing element $g = (\theta^k, n_\alpha e_\alpha)$ of the orbifold space group with

- $SU(3)$ lattice vectors e_1 and e_2
- twist θ (of 120 degrees) with $\theta^3 = 1$

This leads to different classes of closed string states

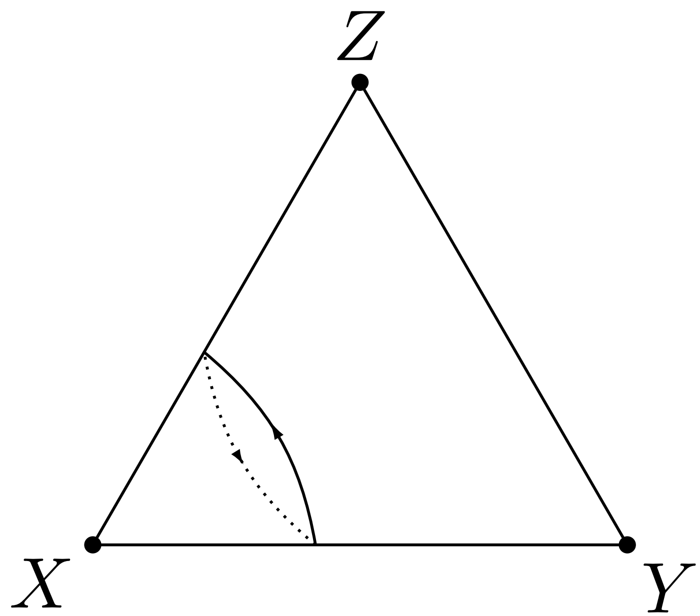
- untwisted states closed on the 2d plane
- winding states $(1, e_i)$ closed on the torus
- twisted states (θ, e_i) closed on the orbifold

How do they transform under $\Delta(54)$ and CP?

Twisted States

While untwisted states transform as singlets, the twisted states transform nontrivially:

- twisted fields $(\theta, 0)$, (θ, e_1) and $(\theta, e_1 + e_2)$ transform as triplets under $\Delta(54)$
- states in the θ^2 sector are anti-triplets
- they wind around fixed points X, Y and Z



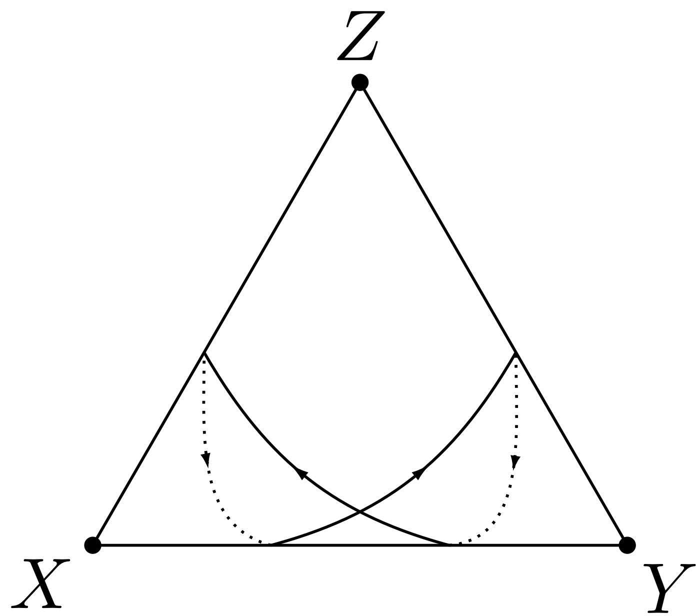
CP maps triplets (anti-triplets) to their complex conjugates

Winding States

Winding states are represented by the geometric elements:

$$V_1 = (1, e_1),\ V_2 = (1, e_2) \text{ and } V_3 = (1, -e_1 - e_2)$$

- the V_i wind around two fixed points with opposite orientation
- winding states $\overline{V}_i$ $i = 1, 2, 3$ have negative winding number



- the geometric winding states V_i and $\overline{V}_i$ do not transform covariantly under $\Delta(54)$

Doublets of $\Delta(54)$

We have to consider linear combinations $[n,\gamma]$

- $$[1,\gamma] = V_1 + \exp(-2\pi i\gamma)V_2 + \exp(-4\pi i\gamma)V_3$$

to obtain covariant states. This leads to doublets of $\Delta(54)$:

- $2_1 = (W_1, \overline{W}_1)$ with $W_1 = [-1, 0]$
- $2_3 = (W_2, \overline{W}_2)$ with $W_2 = \exp(4\pi i/3)[-1, -1/3]$
- $2_4 = (\overline{W}_3, W_3)$ with $W_3 = \exp(2\pi i/3)[-1, 1/3]$

States with positive and negative winding number form the two components of the individual doublets.

Generically, the winding modes are massive. Otherwise we would have symmetry enhancement (Narain lattice).

Examples from MiniLandscape

There are many examples in the heterotic MiniLandscape

(Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, 2006-2008)

- with T_2/Z_3 subsectors
- and potential $\Delta(54)$ symmetry

An inspection of the spectrum reveals that the massless modes transform as

- singlets (untwisted sector)
- triplets (anti-triplets) in θ- (θ^2-) twisted sectors
- there are no doublets in the massless spectrum!!!

For an example see: (Carballo-Perez, Peinado, Ramos-Sanchez, 2016)

CP-symmetry and its violation

We consider CP as a subgroup of S_4 of the outer automorphisms of $\Delta(54)$

- CP transforms (rep) to $(rep)^*$
- this is possible for singlets and triplets
- possible simultaneously for up to two doublets
- impossible in the presence of three or more doublets

The low-energy effective theory allows CP symmetry

- which is broken in the presence of winding modes
- physical CP-violation can arise if there are at least three doublets (here 2_1, 2_3 and 2_4) (Trautner, 2017)

CP-violation in physics

The relevance for physics includes

- CP-violation in the standard model (Jarlskog angle)
- the Θ-parameter of QCD
- CP violation for baryo/lepto-genesis

We have special form of of CP symmetry and CP-violation

- "Explicit geometric CP-violation"
- CP as outer automorphism of flavour symmetry
- CP symmetry for the low energy effective theory broken in the presence of (at least three) $\Delta(54)$ doublets
- Example for "CP made and broken"

Signals of CP-violation

The specific signals of CP-violation are strongly model dependent. We consider as a (toy) example the explicit model of (Carballo-Perez, Peinado, Ramos-Sanchez, 2016)

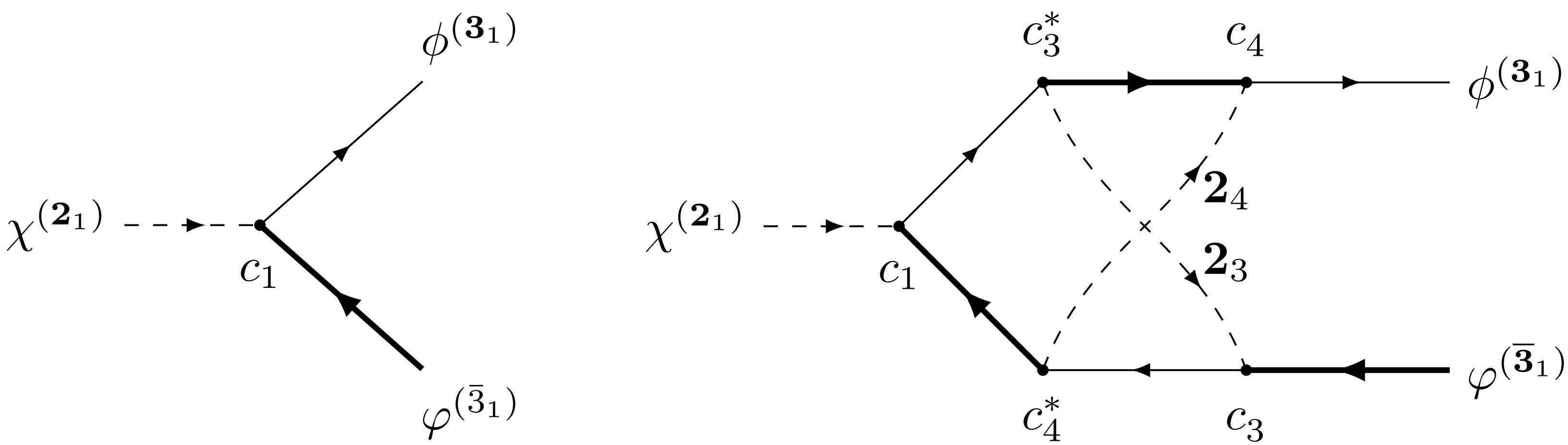
- it contains singlets, triplets and anti-triplets of $\Delta(54)$
- quarks and leptons as triplets
- Higgs as singlet
- right handed neutrinos as anti-triplets
- SM singlets as triplets and anti-triplets of $\Delta(54)$

The relevant couplings to the winding modes 2_i are

- $3 \otimes \bar{3} \to 2_i$ and $3 \otimes 3 \otimes 3 \to 2_i \quad (i = 1, 3, 4)$

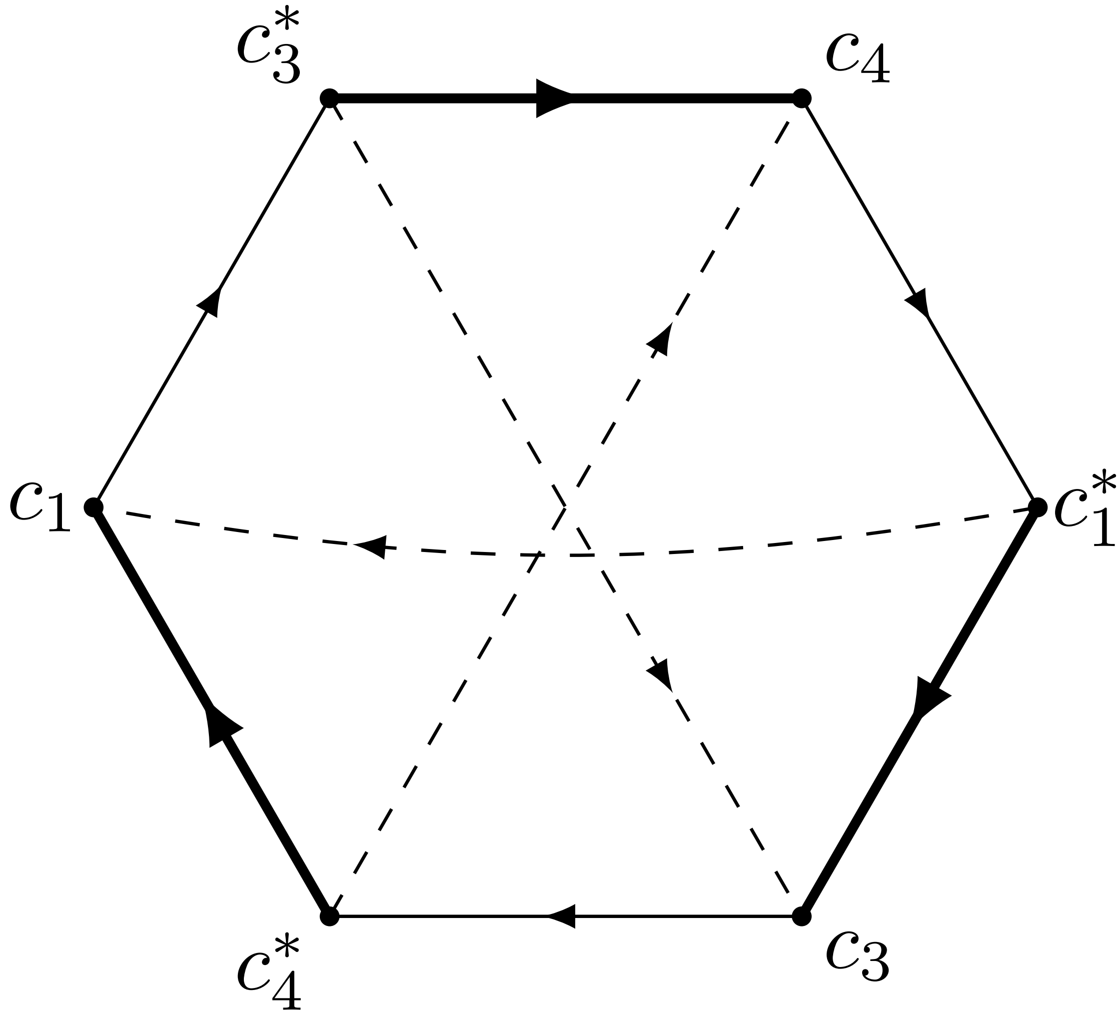
CP violation through decays

CP-violation from the decay of heavy doublets.
All three doublets have to appear in the process.
CP-violation from the interference of two decay diagrams.

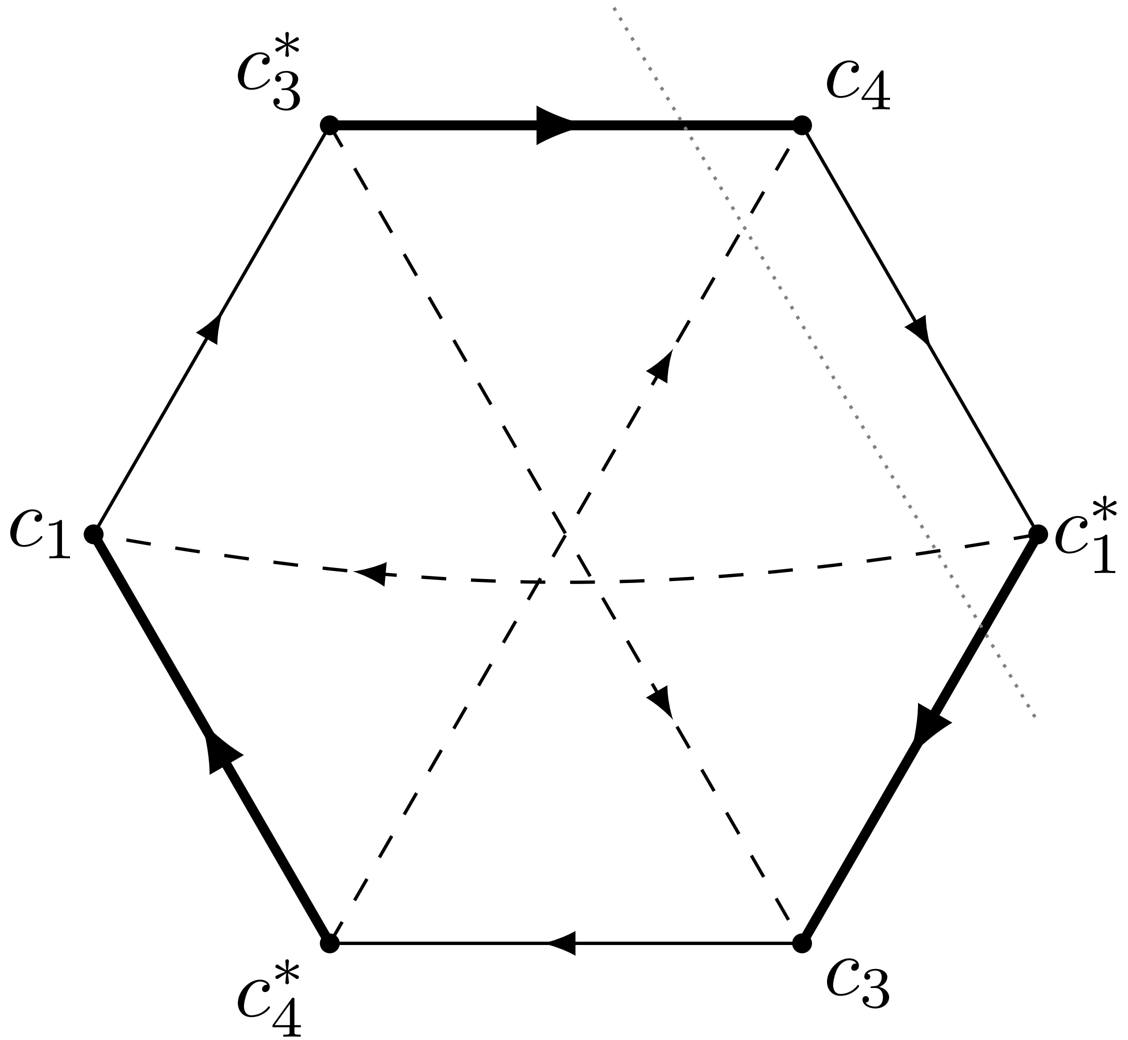


- 2_3 and 2_4 in (non-planar) two-loop diagram
- Decay to right-handed neutrinos and SM singlets as source for lepto-genesis

CP-odd basis invariant



CP-odd basis invariant



CP violation through decays

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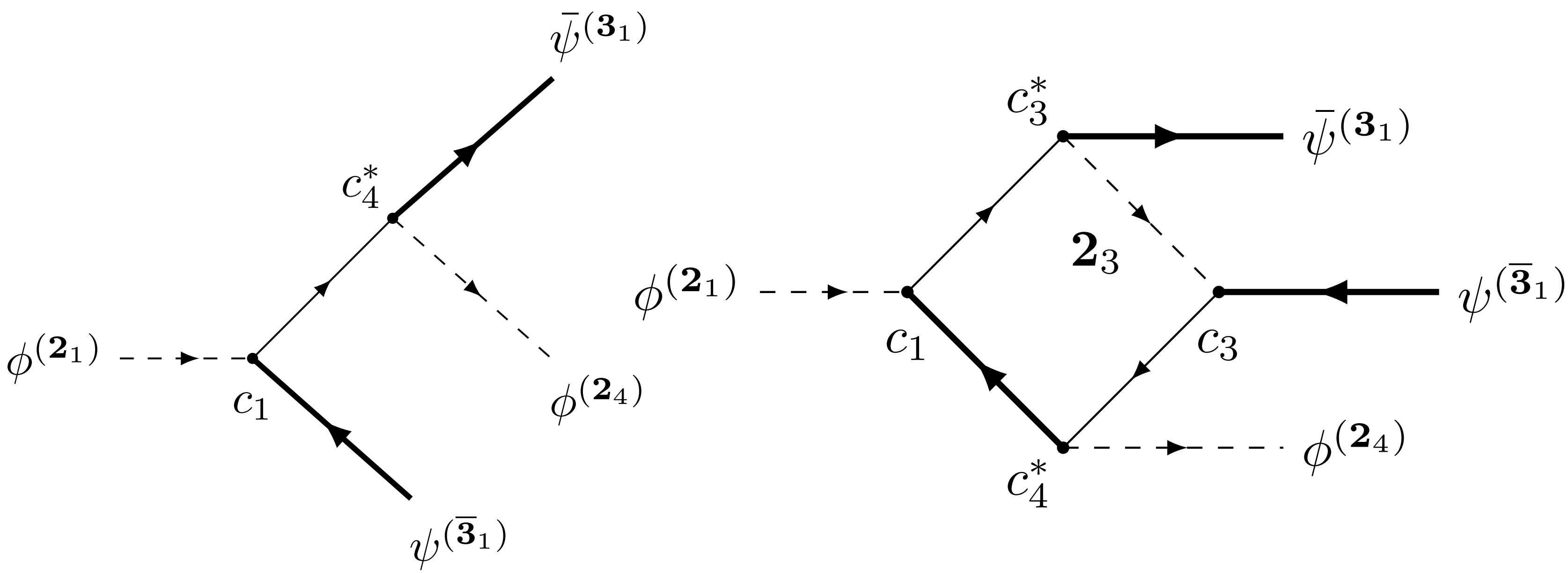
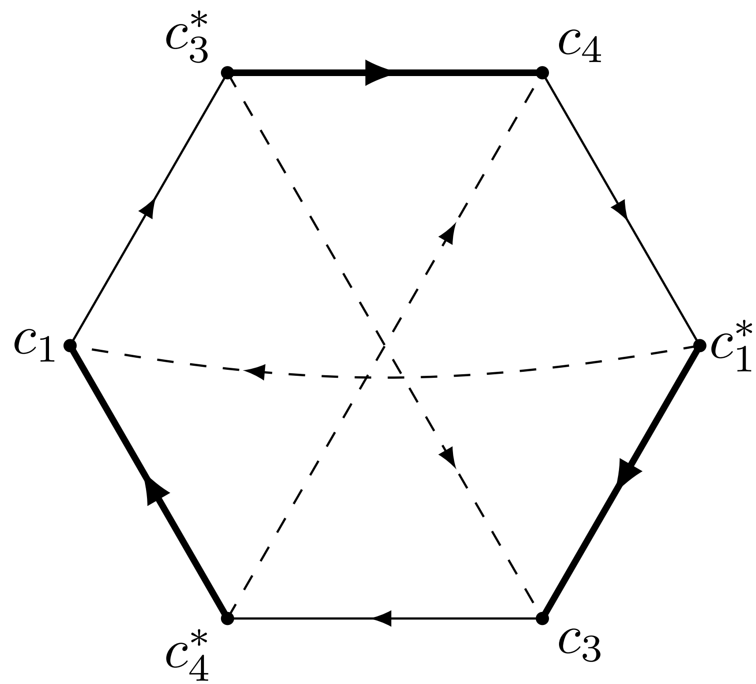


- 2_3 in the one-loop diagram

CP-violation in physics

The "CP-odd basis invariants" control all possible CP-violation in physics

- CP-violating decay of heavy doublets
- CP violation in the standard model (Jarlskog determinant)
- QCD Θ-angle



- We need explicit model building to study these effects (coupling of doublets to CKM matrix and Θ_{QCD})

Conclusions: Part I

Discussion of CP requires

- the origin of the symmetry ("Make It")
- and its violation ("Break It")

String theory could provide such a mechanism through

- "Explicit geometric CP-violation"
- Unification of flavour symmetry and CP
- CP symmetry for the low energy effective theory
- broken in the presence of heavy string modes

It provides calculable effects for CP-violating decay of heavy particles and a solution to the strong CP-problem

II. Search for a general method

We have seen that even in simple systems we obtain sizeable flavor groups

- D_4 for the interval
- $\Delta(54)$ for the 2-dimensional Z_3 orbifold

Did we find the complete flavor symmetry in these cases?

In reality we have even six compact dimensions

- variety of complicated group structures possible
- dependence on localisation of fields in extra dimensions

A general mechanism to find the complete set of flavor symmetries is based on the

outer automorphisms of the Narain space group.

(Baur, Nilles, Trautner, Vaudrevange, 2019)

The Narain Lattice

In the string there are D right- and D left-moving degrees of freedom $Y = (y_R, y_L)$. Y compactified on a $2D$ torus

$$Y = \begin{pmatrix} y_R \\ y_L \end{pmatrix} \sim Y + E\hat{N} = \begin{pmatrix} y_R \\ y_L \end{pmatrix} + E \begin{pmatrix} n \\ m \end{pmatrix}$$

defines the Narain lattice with

- the string's winding and Kaluza-Klein quantum numbers n and m
- the Narain vielbein matrix E that depends on the moduli of the torus: radii, angles and anti-symmetric tensor fields B.

The Narain Space Group

A Z_K orbifold with twist Θ leads to the identification

$$Y \sim \Theta^k Y + E\hat{N} \quad \text{where} \quad \Theta = \begin{pmatrix} \theta_R & 0 \\ 0 & \theta_L \end{pmatrix} \quad \text{and} \quad \Theta^K = 1$$

with θ_L, θ_R elements of $SO(D)$. For a symmetric orbifold $\theta_L = \theta_R$ (we do not include roto-translations here).

The Narain space group $g = (\Theta^k, E\hat{N})$ is then generated by

$$\text{twists} \quad (\Theta, 0) \quad \text{and shifts} \quad (1, E_i) \quad \text{for} \quad i = 1 \ldots 2D$$

Outer automorphisms map the group to itself but are not elements of the group.

Duality Transformations

Modular transformations (dualities) exchange windings and momenta and act nontrivially on the moduli of the torus.

In $D = 2$ these transformations are connected to the group $SL(2, Z)$ acting on Kähler and complex structure moduli.

The group $SL(2, Z)$ is generated by two elements

$$S \ , \ T : \qquad \text{with} \ \ S^4 = 1 \ \ \text{and} \ \ S^2 = (ST)^3$$

On a modulus M with have the transformations

$$S : \quad M \to -\frac{1}{M} \quad \text{and} \quad T : M \to M + 1$$

Further transformations might include $M \to -\overline{M}$ and mirror symmetry between Kähler and complex structure moduli.

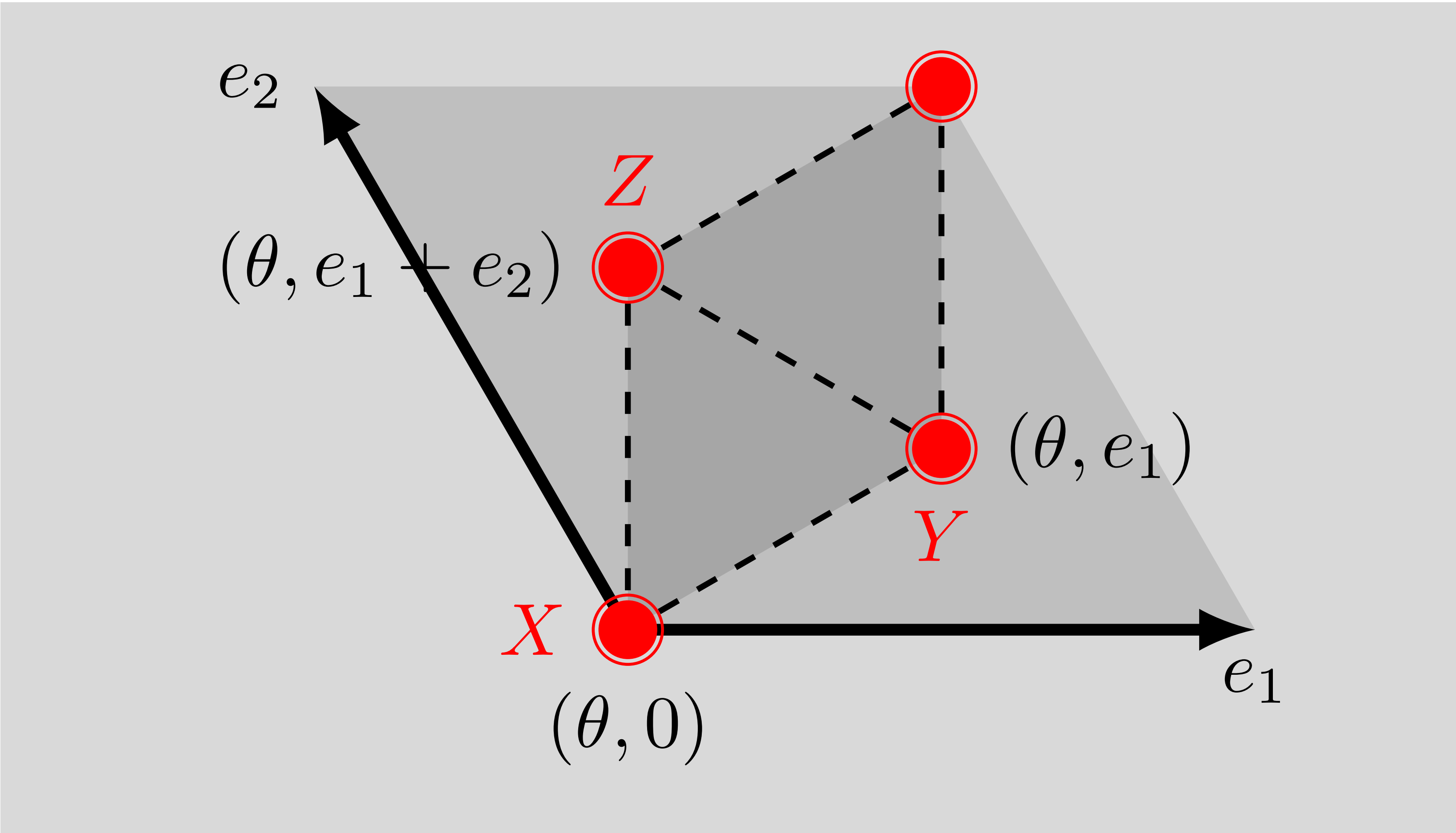
Candidate symmetries

As outer automorphisms of the Narain space group we can identify

- traditional flavor symmetries which are universal in moduli space
- a subset of the modular transformations that act as symmetries at specific "points" in moduli space
- at these "points" we shall have an enhanced symmetry that combines the traditional flavor symmetry with some of the modular symmetries

The full flavor symmetry is non-universal in moduli space.
At generic points in moduli space we have the universal traditional flavor symmetry.

Orbifold T_2/Z_3



Example: T_2/Z_3 Orbifold

On the orbifold some of the moduli are frozen

- lattice vectors e_1 and e_2 have the same length
- angle is 120 degrees

Modular transformations form a subgroup of $SL(2,Z)$

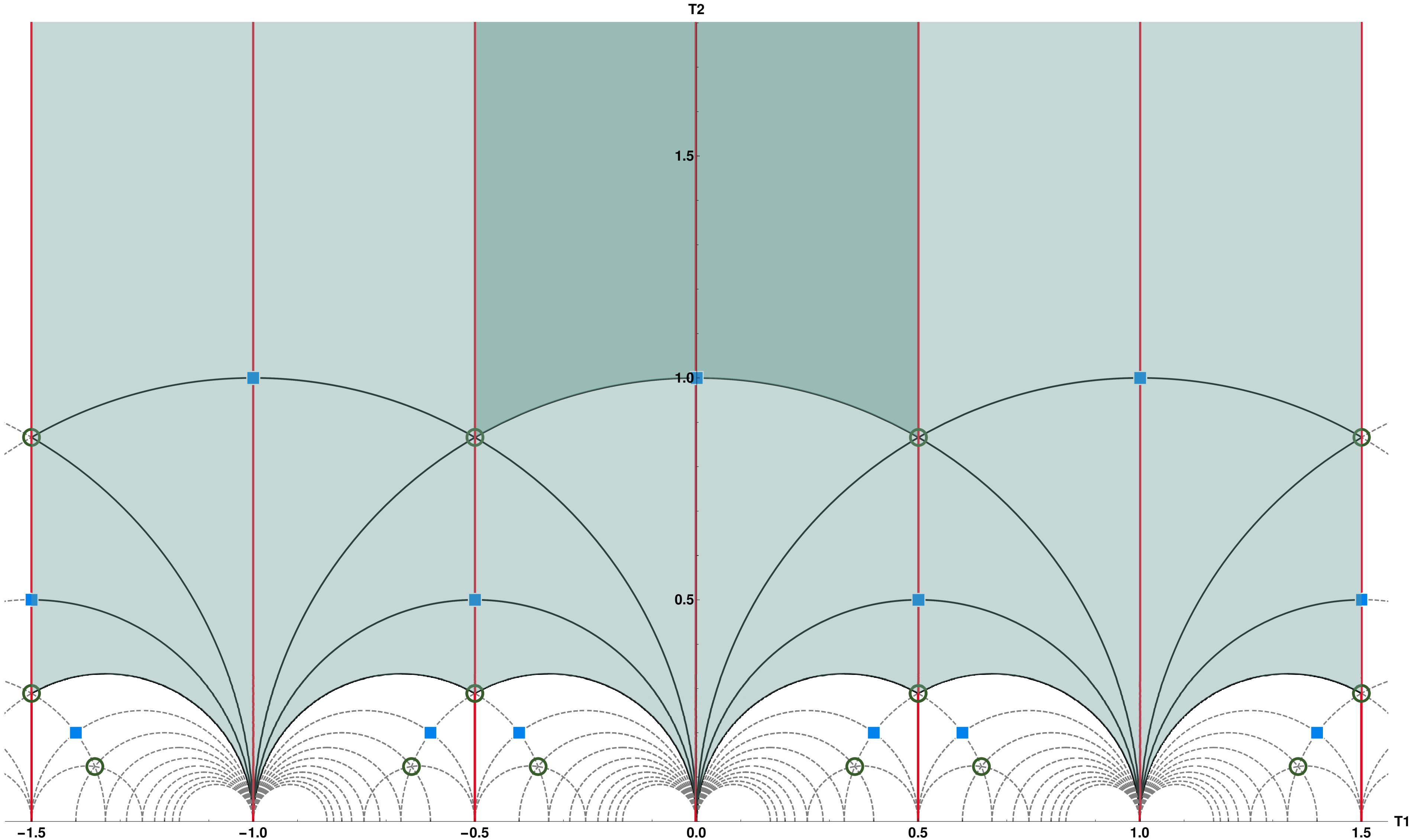
- $\Gamma(3)$ as a mod(3) subgroup of $SL(2,Z)$; ($\Gamma(3) = A_4$)
- $\Gamma(3)$ acts on the moduli
- twisted fields transform under a bigger group T', (similar to enhancement of $SO(3)$ to $SU(2)$ for spinors)

(Lauer, Mas, Nilles, 1989; Lerche, Lüst, Warner, 1989)

- transformation $M \to -\overline{M}$ completes the picture

Full group is SG(48,29) with 48 elements

Moduli space of $\Gamma(3)$



Flavor Symmetries I

Generic point in moduli space.

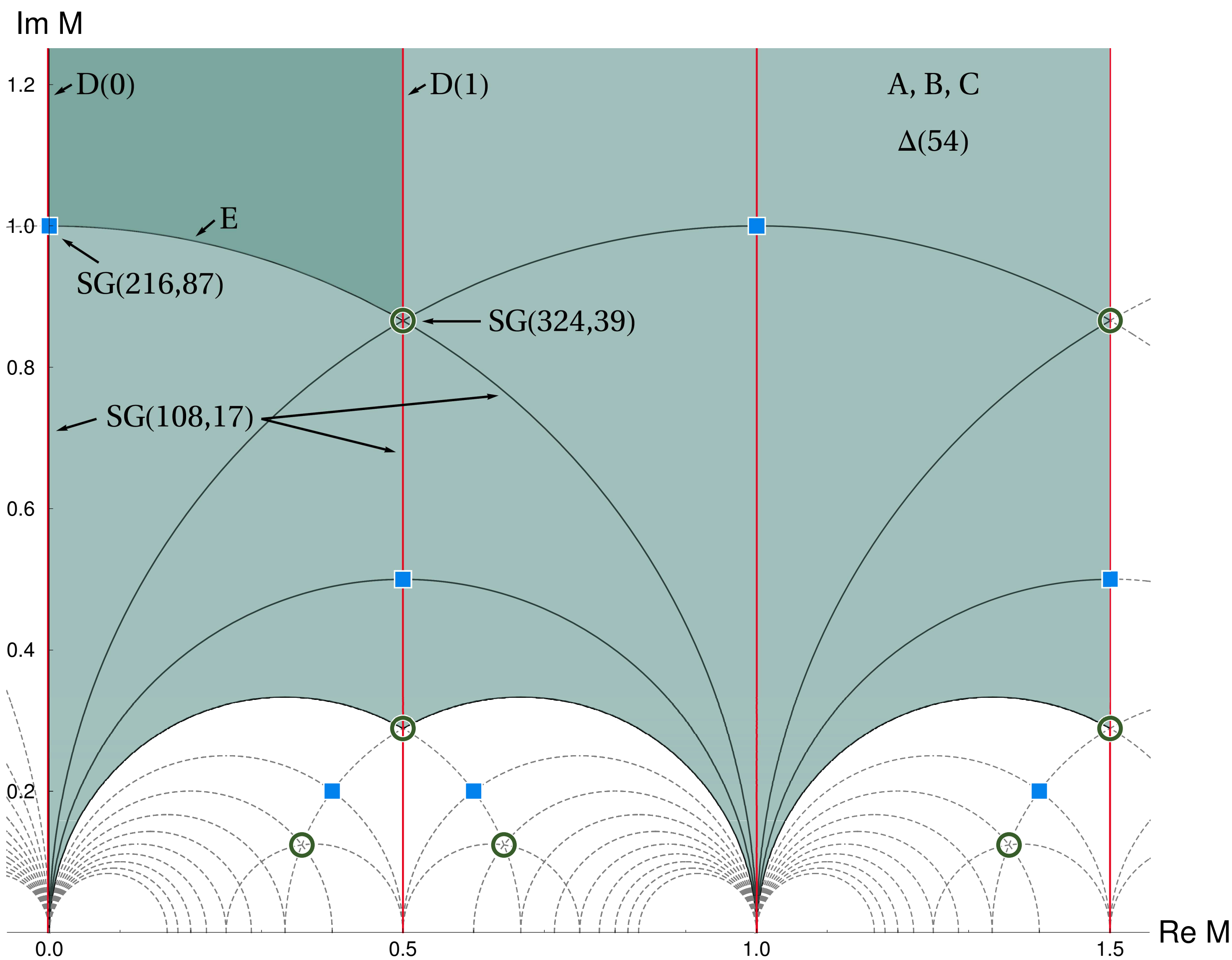
Outer automorphisms of the Narain space group are

- shift $A = (1_4; \frac{1}{3}, \frac{2}{3}, 0, 0)$
- and shift $B = (1_4; 0, 0, \frac{1}{3}, \frac{1}{3})$
- a left-right symmetric rotation $C = (-1_4; 0, 0, 0, 0)$

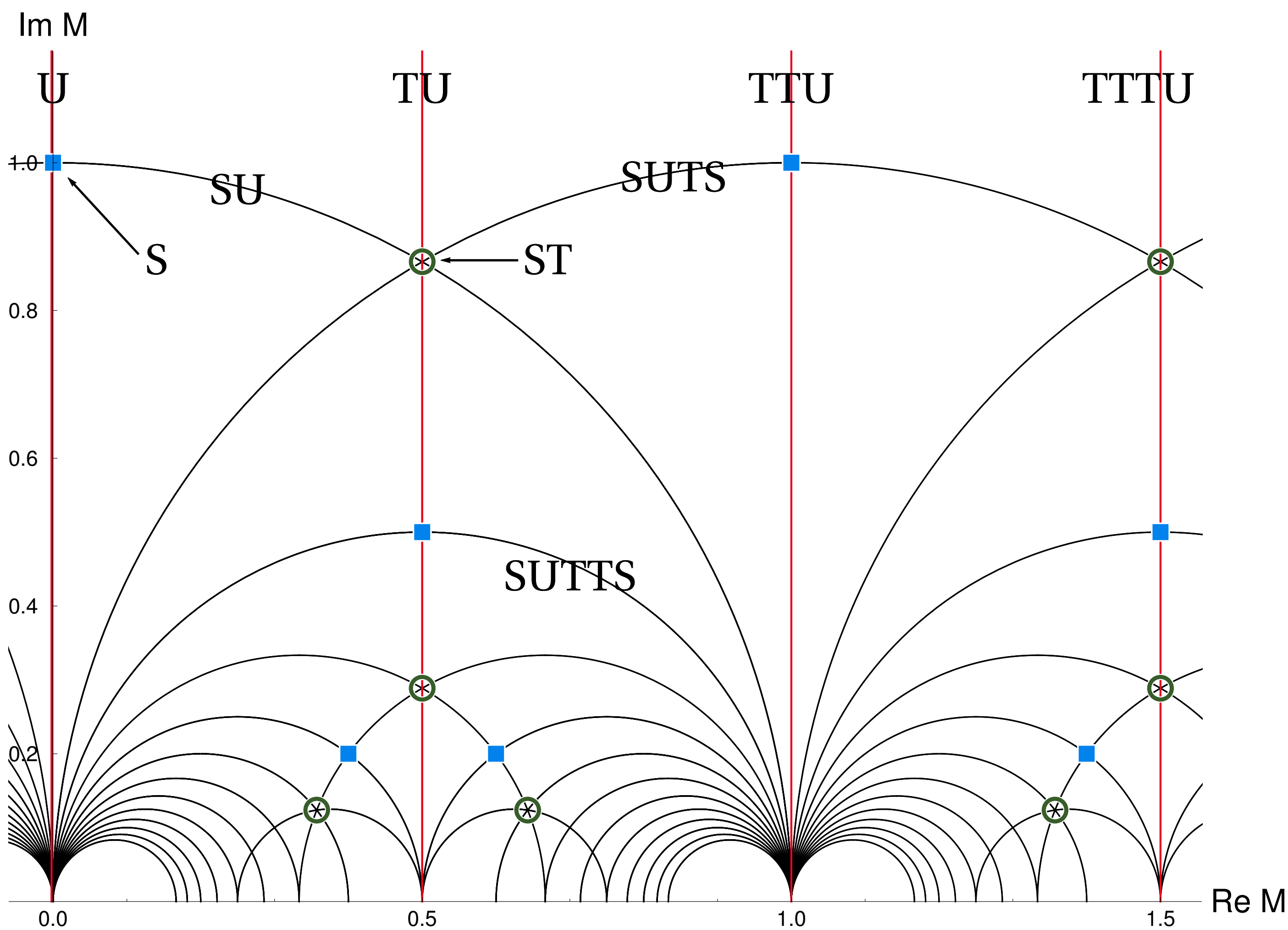
Multiplicative closure of A, B and C leads to $\Delta(54)$.

- the earlier guesswork gave the correct result!
- but the new method produces the result automatically
- can be generalised easily to more complicated situations (like, e.g. six dimensions)

Moduli space of flavour groups



Fixed lines and points



$$S: M \to -\frac{1}{M}, \quad T: M \to M+1 \quad \text{and} \quad U: M \to -\overline{M}$$

Flavor Symmetries II

Duality transformations might become symmetries!

The red lines:

These are fixed lines under T and U. We have

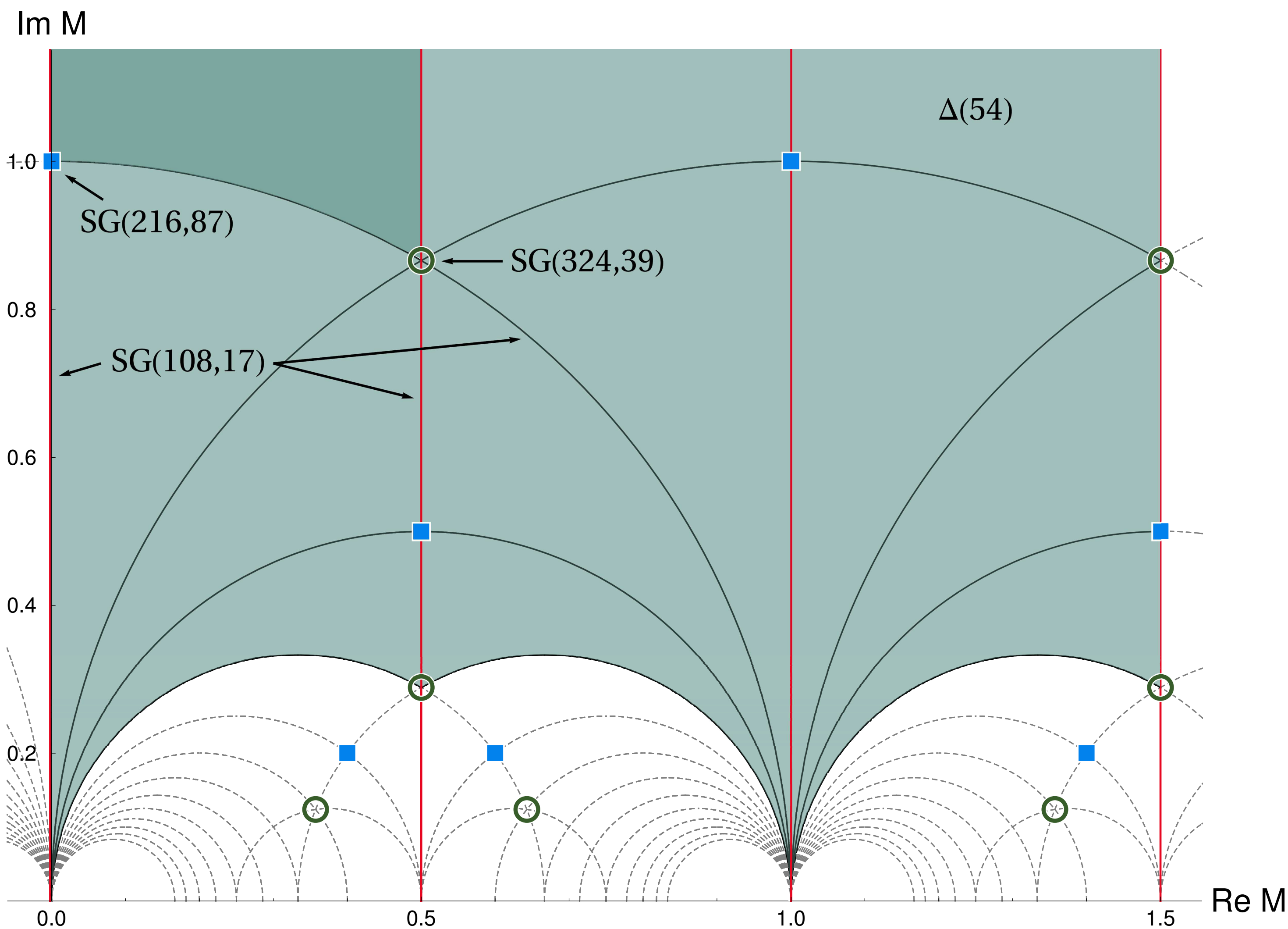
- again A, B, C and a left-right symmetric reflection D

Multiplicative closure leads to $SG(108, 17)$. This includes the formerly discussed CP-transformation! Unification of flavor and CP (spontaneous breakdown away from the line).

The circles: e.g. fixed lines under S and U

- new asymmetric reflection E (instead of D)
- again $SG(108, 17)$ but differently aligned
- enhanced with different Z_2 from $S_4 = \mathrm{Out}(\Delta(54))$

Moduli space of flavour groups



Flavor Symmetries III

Blue squares: two lines meet

- enhancement to $SG(216, 87)$

The small circles: three lines meet

- maximum enhancement to $SG(324, 39)$

The modular group T' has 24 elements, but not all of them lead to an enhancement of the flavor group $\Delta(54)$.

Only the elements within S_4 of the outer automorphisms of $\Delta(54)$ are relevant

- this leads to unification of flavour and CP
- CP exact at those fixed lines and points

Messages

We have designed a generic method to find all flavor symmetries (based on the Narain space group)

- unification of traditional (discrete) flavor, CP and modular symmetries
- traditional flavor symmetry is universal in moduli space
- there are non-universal enhancements (including CP at some places (broken in generic moduli space))
- CP is a consequence of the duality symmetries of string theory
- the potential flavor groups are large and non-universal (in our example already up to $SG(324, 39)$ for two extra dimensions)

Consequences

This opens a new arena for flavor model building

- a new look at CP as discrete gauge symmetry

  (Nilles, Ratz, Trautner, Vaudrevange, 2018)

- modular symmetries for flavor (Altarelli, Feruglio, 2006; Feruglio, 2017)
- groups are large and allow for flexibility (Hagedorn, König, 2018)
- the concept of local flavor symmetries allows different flavor groups for different sectors of the theory

  (Baur, Nilles, Trautner, Vaudrevange, 2019)

- non-universal structure from duality symmetries (there is still the traditional universal flavor group)
- different flavor symmetries for quarks and leptons are no surprise

Sorak1984

Sorak1984