

# Unification of Flavor, CP and Modular Symmetries

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# Outline

We discuss the world of flavor from a **top-down** point of view derived from string theory. It relies heavily on the

- Narain lattice formalism for torus compactification
- the **Narain space group** of the twisted lattice

Discrete flavor symmetries are identified as the **outer automorphisms of the Narain space group**. They include

- Traditional flavor symmetries
- Modular flavor symmetries
- Natural candidate for a CP-symmetry

(Baur, Nilles, Trautner, Vaudrevange, 2019)

leading to a unified picture of discrete flavor symmetries.

# Eclectic Flavor Group

These symmetries combine to the so-called **eclectic flavor group**, including (Nilles, Ramos-Sanchez, Vaudrevange, 2020)

- Traditional flavor symmetries, CP symmetry and
- Modular flavor symmetries

An important aspect is the appearance of the discrete symmetries derived from the **modular and symplectic groups  $SL(2, \mathbb{Z})$  and  $Sp(4, \mathbb{Z})$**  which appear in compactifications that exhibit elliptic fibrations. This leads to

- **A unified picture of quark- and lepton-flavor: localized flavor groups**

(Baur, Kade, Nilles, Ramos-Sanchez, Vaudrevange, 2020)

# Discrete Flavor Symmetries

Most of the parameters of the SM concern flavor physics: masses and mixing angles of quarks and leptons.

- Many fits from **bottom-up** perspective with discrete symmetries ( $S_3$ ,  $A_4$ ,  $S_4$ ,  $A_5$ ,  $\Delta(27)$ ,  $\Delta(54)$  etc.)
- **Flavor symmetries seem to require different models for quark and lepton sector** (small mixing angles for quarks versus large mixing in lepton sector)
- Flavor symmetries are spontaneously broken. This requires the introduction of **so-called flavon fields** and additional parameters
- bottom-up model building leads to **successful** fits for various choices of groups and representations

**But we are still missing a top-down explanation of flavor**

# Flavor Symmetries in String Theory

Such a top-down explanation might come from string theory. Discrete symmetries might have their origin in

- specific string theory selection rules
- symmetries of compact extra dimensions.

As an illustration we consider orbifold compactifications of the heterotic string. They provide:

- the chiral spectrum of quarks and leptons at low energies within the  $SU(3) \times SU(2) \times U(1)$  standard model
- abundant discrete symmetries as candidates for flavor symmetries that might allow a connection to existing bottom-up constructions of flavor

# Unification of flavor and CP

Summary of predictions of the string picture:

- traditional flavor symmetries (**universal** in moduli space)
- modular flavor symmetries and CP are **non-universal** in moduli space

They unify in the **eclectic picture** of flavor symmetry.  
You cannot just have one without the other.

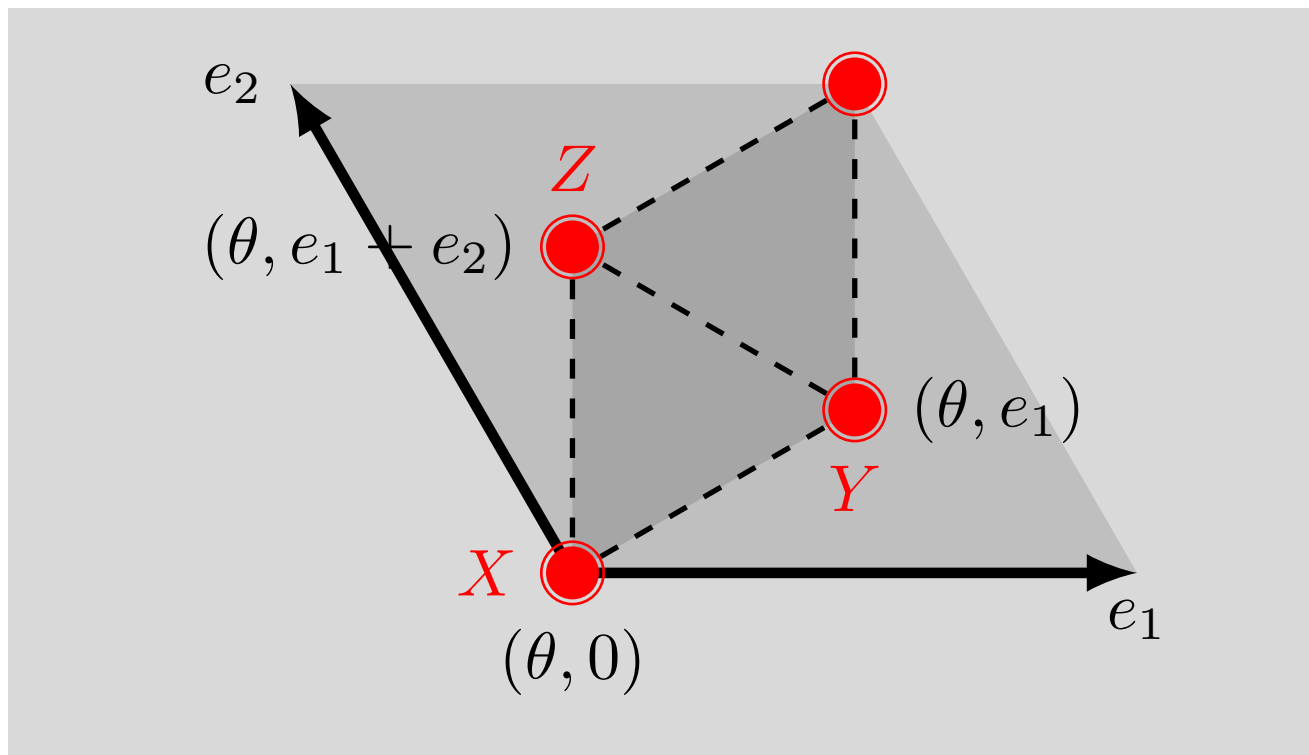
The non-universality in moduli space leads to

- local flavor unification at specific points in moduli space
- **potentially different pictures** for quarks and leptons
- **spontaneous breakdown** of flavor and CP as a **motion in moduli space**

# Flavor from strings

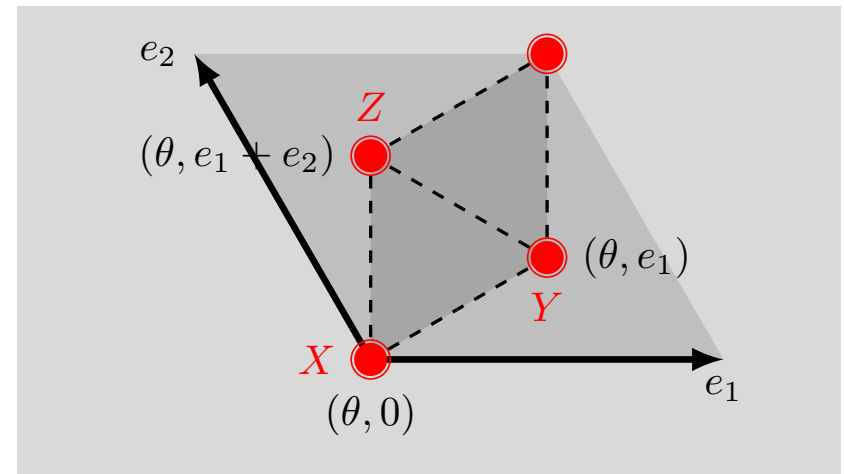
In string theory discrete symmetries can arise from geometry and string selection rules.

As an example we consider the orbifold  $T_2/Z_3$



# Discrete symmetry $\Delta(54)$

- untwisted and twisted fields
- $S_3$  symmetry from interchange of fixed points
- $Z_3 \times Z_3$  symmetry from orbifold selection rules



- $\Delta(54)$  as multiplicative closure of  $S_3$  and  $Z_3 \times Z_3$
- $\Delta(54)$  – a non-abelian subgroup of  $SU(3)_{\text{flavor}}$
- e.g. flavor symmetry for three families of quarks (as triplets of  $\Delta(54)$ )

(Kobayashi, Nilles, Ploeger, Raby, Ratz, 2006)



# Search for a general method

We have seen that even in simple systems we obtain sizeable flavor groups

- $\Delta(54)$  for the 2-dimensional  $Z_3$  orbifold sector

Did we find the complete flavor symmetry in these cases?

In reality we have even six compact dimensions

- variety of complicated group structures possible
- dependence on localisation of fields in extra dimensions

A general mechanism to find the complete set of flavor symmetries is based on the

outer automorphisms of the Narain space group.

(Baur, Nilles, Trautner, Vaudrevange, 2019)

# The Narain Lattice

In the string there are  $D$  right- and  $D$  left-moving degrees of freedom  $Y = (y_R, y_L)$ .  $Y$  compactified on a  $2D$  torus

$$Y = \begin{pmatrix} y_R \\ y_L \end{pmatrix} \sim Y + E\hat{N} = \begin{pmatrix} y_R \\ y_L \end{pmatrix} + E \begin{pmatrix} n \\ m \end{pmatrix}$$

defines the  $2D$  **Narain lattice** (4-dim for 2-torus ( $D = 2$ )) with

- the string's **winding and Kaluza-Klein quantum numbers**  $n$  and  $m$  (respectively 2-dim. for 2-torus)
- the **Narain vielbein matrix**  $E$  that depends on the moduli of the torus: radii, angles and anti-symmetric tensor fields  $B$ .

# The Narain Space Group

A  $Z_K$  orbifold with twist  $\Theta$  leads to the identification

$$Y \sim \Theta^k Y + E\hat{N} \quad \text{where} \quad \Theta = \begin{pmatrix} \theta_R & 0 \\ 0 & \theta_L \end{pmatrix} \quad \text{and} \quad \Theta^K = 1$$

with  $\theta_L, \theta_R$  elements of  $SO(D)$ . For a symmetric orbifold  $\theta_L = \theta_R$  (we do not include roto-translations here).

The Narain space group  $g = (\Theta^k, E\hat{N})$  is then generated by

twists  $(\Theta, 0)$  and shifts  $(1, E_i)$  for  $i = 1 \dots 2D$

Outer automorphisms map the group to itself but are not elements of the group (include modular transformations).

# T-duality in String Theory

This interplay of momentum and winding modes is the origin of T-duality when we simultaneously interchange

- momentum  $\rightarrow$  winding
- $R \rightarrow 1/R$

This transformation maps a theory to its T-dual theory.

- self-dual point is  $R^2 = \alpha' = 1/M_{\text{string}}^2$

If the string scale  $M_{\text{string}}$  is large then the low energy effective theory describes the momentum states and the winding states are heavy.

Can T-duality play a relevant role for flavor symmetry?

# Duality Transformations

Modular transformations (dualities) exchange windings and momenta and act nontrivially on the moduli of the torus.

In  $D = 2$  these transformations are connected to the group  $SL(2, Z)$  acting on Kähler and complex structure moduli.

The group  $SL(2, Z)$  is generated by two elements

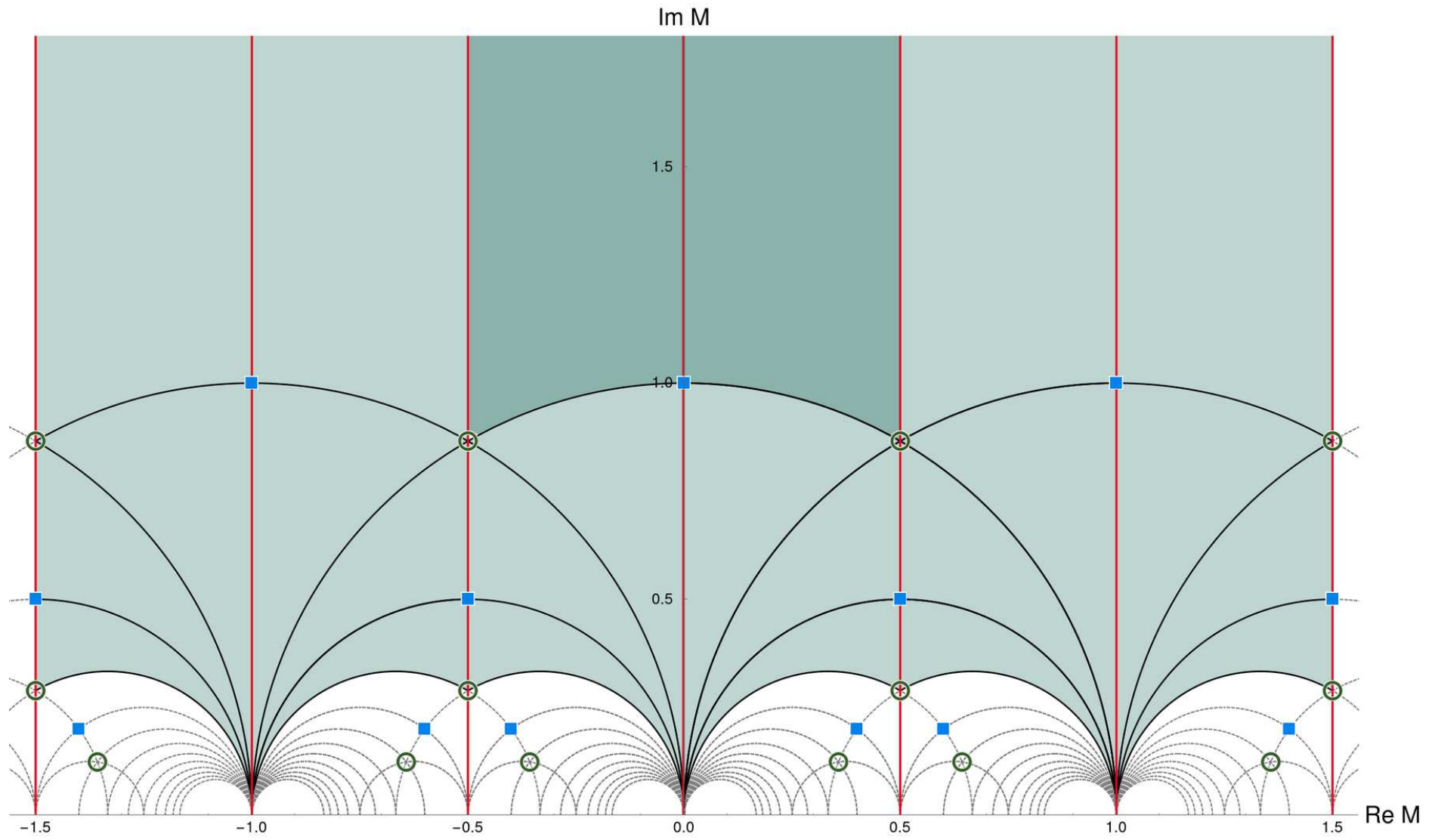
$$S, T : \quad \text{with } S^4 = 1 \text{ and } S^2 = (ST)^3$$

A modulus  $M$  transforms as

$$S : M \rightarrow -\frac{1}{M} \quad \text{and} \quad T : M \rightarrow M + 1$$

Further transformations might include  $M \rightarrow -\overline{M}$  and mirror symmetry between Kähler and complex structure moduli.

# Fundamental Domain



# Modular Forms

String dualities give important constraints on the action of the theory via the **modular group**  $SL(2, Z)$ :

$$\gamma : M \rightarrow \frac{aM + b}{cM + d}$$

with  $ad - bc = 1$  and integer  $a, b, c, d$ .

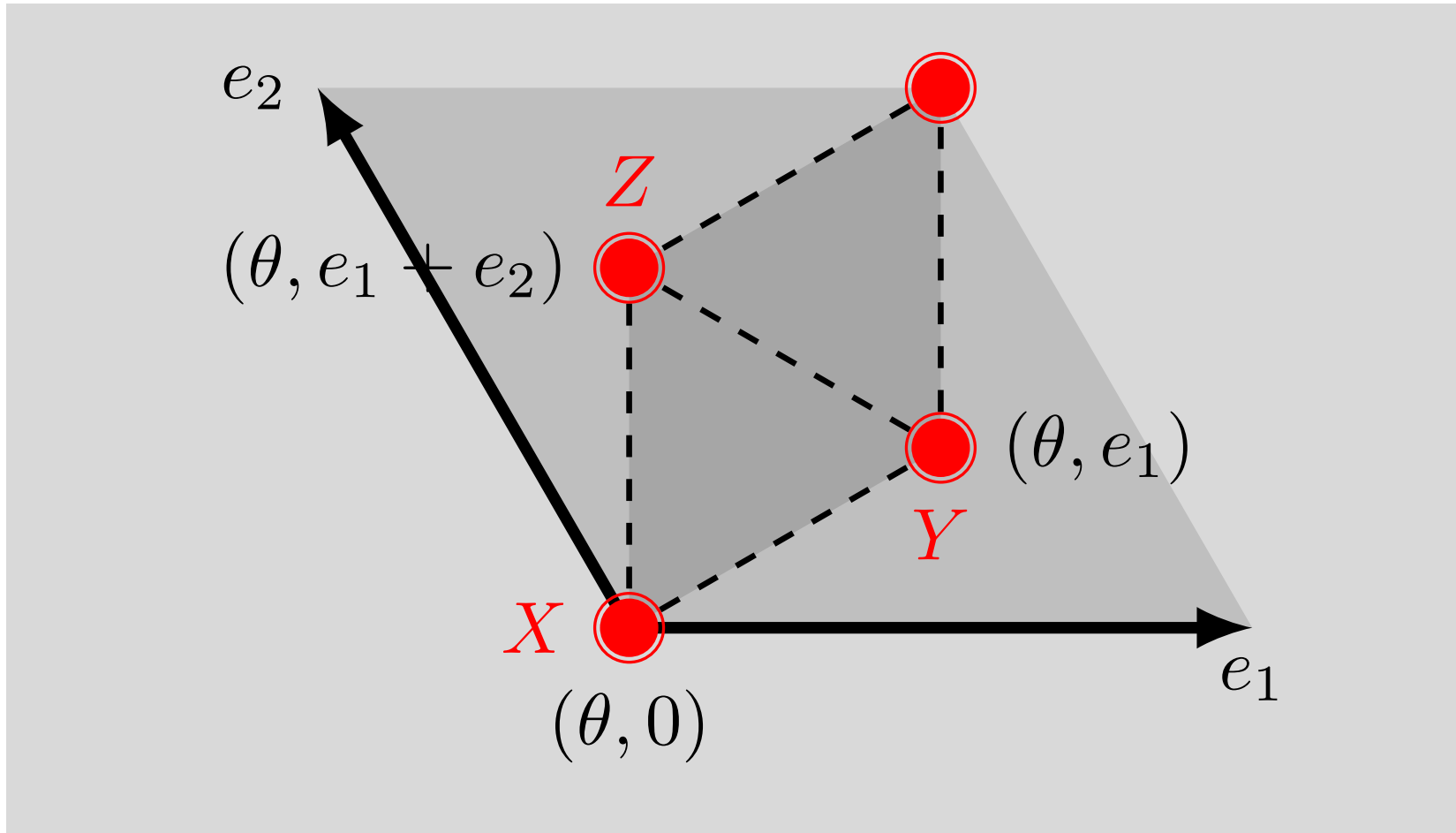
Matter fields transform as **modular functions** of weight  $k$

$$\gamma : \phi \rightarrow (cM + d)^k \rho(\gamma) \phi .$$

**Yukawa-couplings** transform as modular functions as well.

$G = K + \log |W|^2$  must be invariant under T-duality

# Orbifold $T_2/Z_3$





# Example: $T_2/Z_3$ Orbifold

On the orbifold some of the moduli are frozen

- lattice vectors  $e_1$  and  $e_2$  have the same length
- angle is 120 degrees

Modular transformations form a subgroup of  $SL(2, Z)$

- $\Gamma(3)$  as a mod(3) subgroup of  $SL(2, Z)$
- **discrete modular flavor group**  $\Gamma_3 = SL(2, Z)/\Gamma(3)$
- here the full discrete modular group is not just  $\Gamma_3 \sim A_4$  but its **double cover**  $T' \sim SL(2, 3)$  (which acts nontrivially on the twisted fields)
- the CP transformation  $M \rightarrow -\overline{M}$  completes the picture.

**Full discrete modular group is  $GL(2, 3) = [48, 29]$ .**

# Flavor Symmetries

As outer automorphisms of the Narain space group we can identify

- traditional flavor symmetries which are **universal in moduli space**
- a subset of the modular **transformations** that act as **symmetries** at specific "points" in moduli space
- at these "points" we shall have an **enhanced symmetry** that combines the traditional flavor symmetry with some of the modular symmetries

**The full flavor symmetry is non-universal in moduli space.**  
At generic points in moduli space we have the universal traditional flavor symmetry.

# Eclectic Flavor Groups

We have thus two types of flavor groups

- the **traditional flavor group** that is universal in moduli space (here  $\Delta(54)$ )
- the **modular flavor group** that transforms the moduli nontrivially (here  $T'$ )

The **eclectic flavor group** is defined as the multiplicative closure of these groups. Here we obtain for  $T_2/Z_3$

- $\Omega(1) = [648, 533]$  from  $\Delta(54)$  and  $T' = SL(2, 3)$
- $[1296, 2891]$  from  $\Delta(54)$  and  $GL(2, 3)$  including CP

The eclectic group is the largest possible flavor group for the given system, **but it is not necessarily linearly realized.**

# Modular Flavor Symmetries

Recently, Feruglio has pioneered the consideration of discrete modular flavor symmetry from a **bottom-up** approach

(Feruglio, 2017)

- His model was based on the **discrete modular group**  $\Gamma_3$  which is isomorphic to  $A_4$
- By now there are many more constructions based on  $\Gamma_N$  for higher  $N$  (e.g.  $\Gamma_4 = S_4$ ,  $\Gamma_5 = A_5$ )
- **Bottom-up** model building includes the choices of
  - specific **representations of**  $\Gamma_N$  and
  - the values of **modular weights of matter fields**

Here we would like to study modular flavor symmetries from the **top-down** perspective to obtain a UV-completion.

# Modular Flavor Symmetries

Modular flavor symmetries from the top-down approach are very restrictive with high predictive power.

- **flavon fields** given by moduli
- modular flavor gives **discrete group**  $\Gamma_N$
- **the values of modular weights are fixed in string theory** and might enhance this symmetry further (origin of "shaping symmetries")
- **always accompanied by a traditional flavor symmetry** (which further restricts the couplings in Kähler- and superpotential)

The (eclectic) top-down construction reflects the symmetries of the underlying string theory

# The "Representation Dilemma"

In the bottom-up approach for the modular group  $\Gamma_3 \sim A_4$

- one assigns e.g. triplet  $3$  and (non-)trivial singlet representations  $1, 1', 1''$  for left-handed and right-handed leptons respectively
- there is complete freedom in choice of modular weights

Compared to the top-down approach based on the group  $T'$  (the double cover of  $\Gamma_3$ )

- twisted fields transform as  $2' + 1$  (not as triplet)
- in addition only trivial singlets in the low energy sector
- modular weights of light fields restricted to  $-2/3, -5/3$

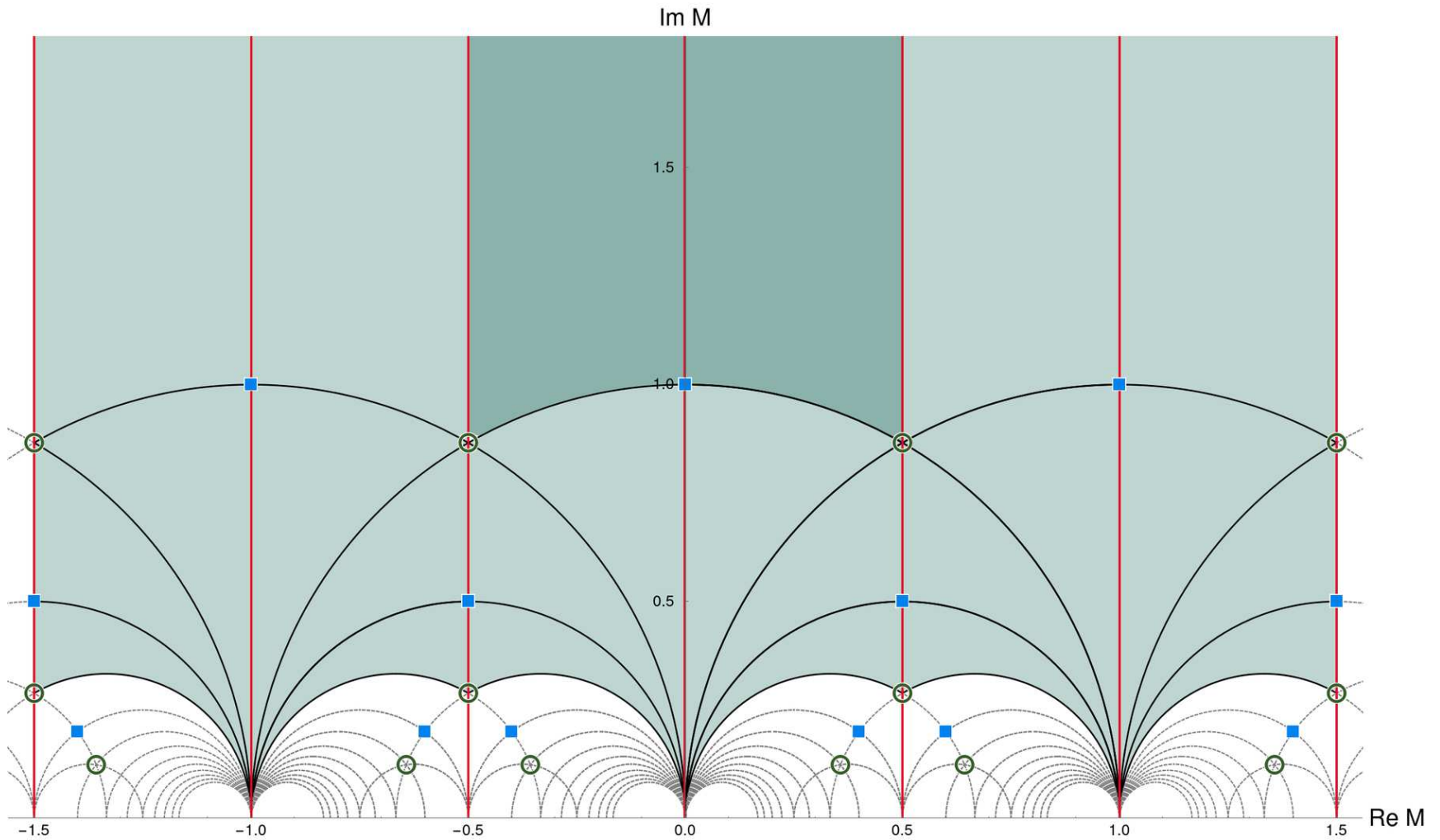
It might be that many of the bottom-up models fail to have a consistent UV-completion.

# Where are we?

So far we have discussed the 2-dim.  $Z_3$  orbifold with the Kähler modulus (usually called  $T$ ). More generally we have to consider the complex structure modulus  $U$  as well:

- this leads to  $SL(2, Z)_T \times SL(2, Z)_U$
- $U$  is frozen in the  $Z_3$  case, but still contributes to the eclectic flavor symmetry with  $R$ -symmetries, extending  $[648, 533] = \Omega(1)$  to  $[1944, 3448] = \Omega(2)$  without  $CP$   
(Nilles, Ramos-Sanchez, Vaudrevange, 2020)
- in the  $Z_2$  case both types of moduli contribute and lead to an eclectic group with 2304 elements (excluding  $CP$ )  
(Baur, Kade, Nilles, Ramos-Sanchez, Vaudrevange, 2020)
- Siegel modular group  $Sp(4, Z)$  including a Wilson-line  
(Ding, Feruglio, Liu, 2020; Baur, Kade, Nilles, Ramos-Sanchez, Vaudrevange, 2020;  
see also talk of A. Font at StringPheno 2021 for background material)

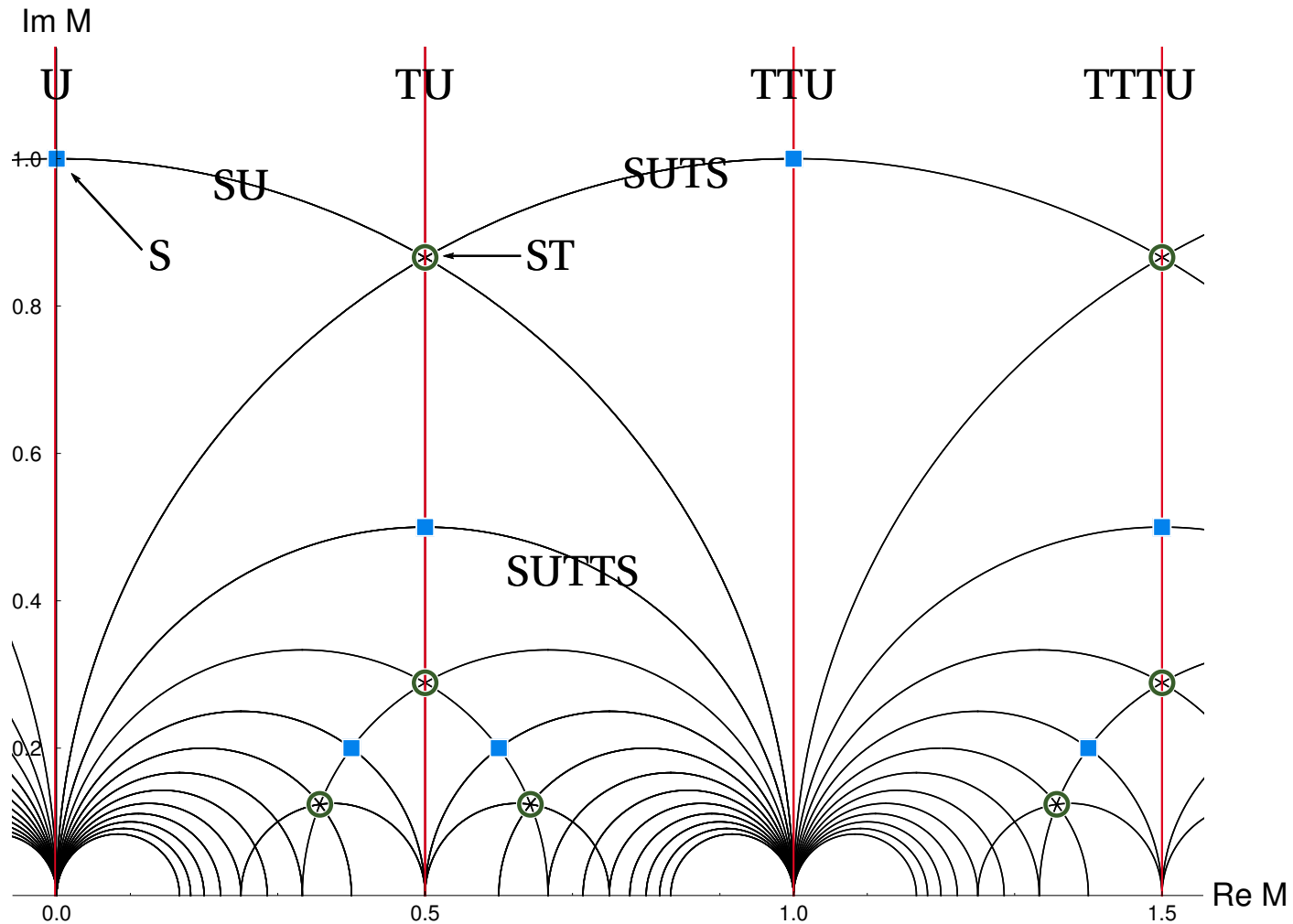
# Local Flavor Unification



Moduli space of  $\Gamma_3$

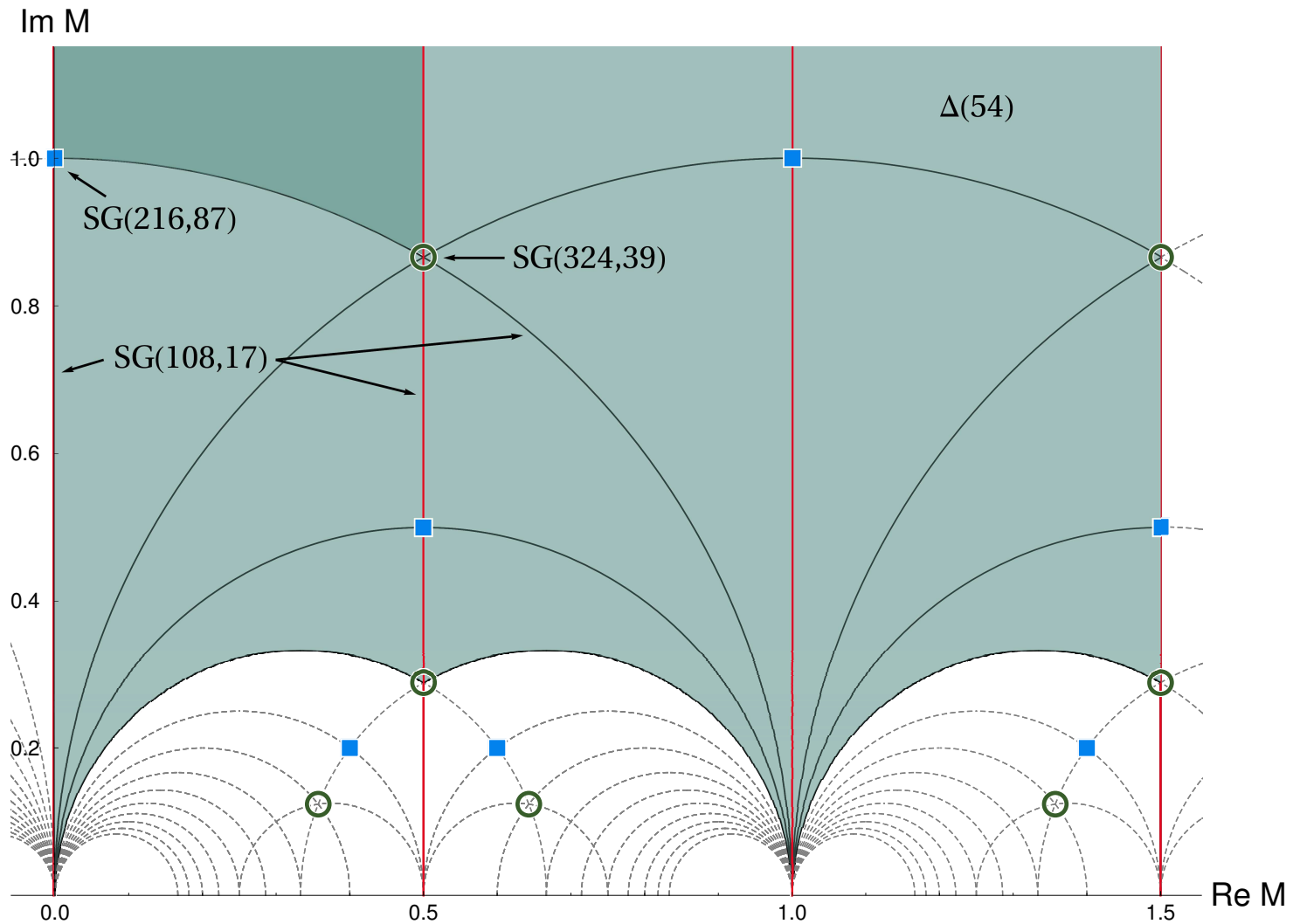


# Fixed lines and points

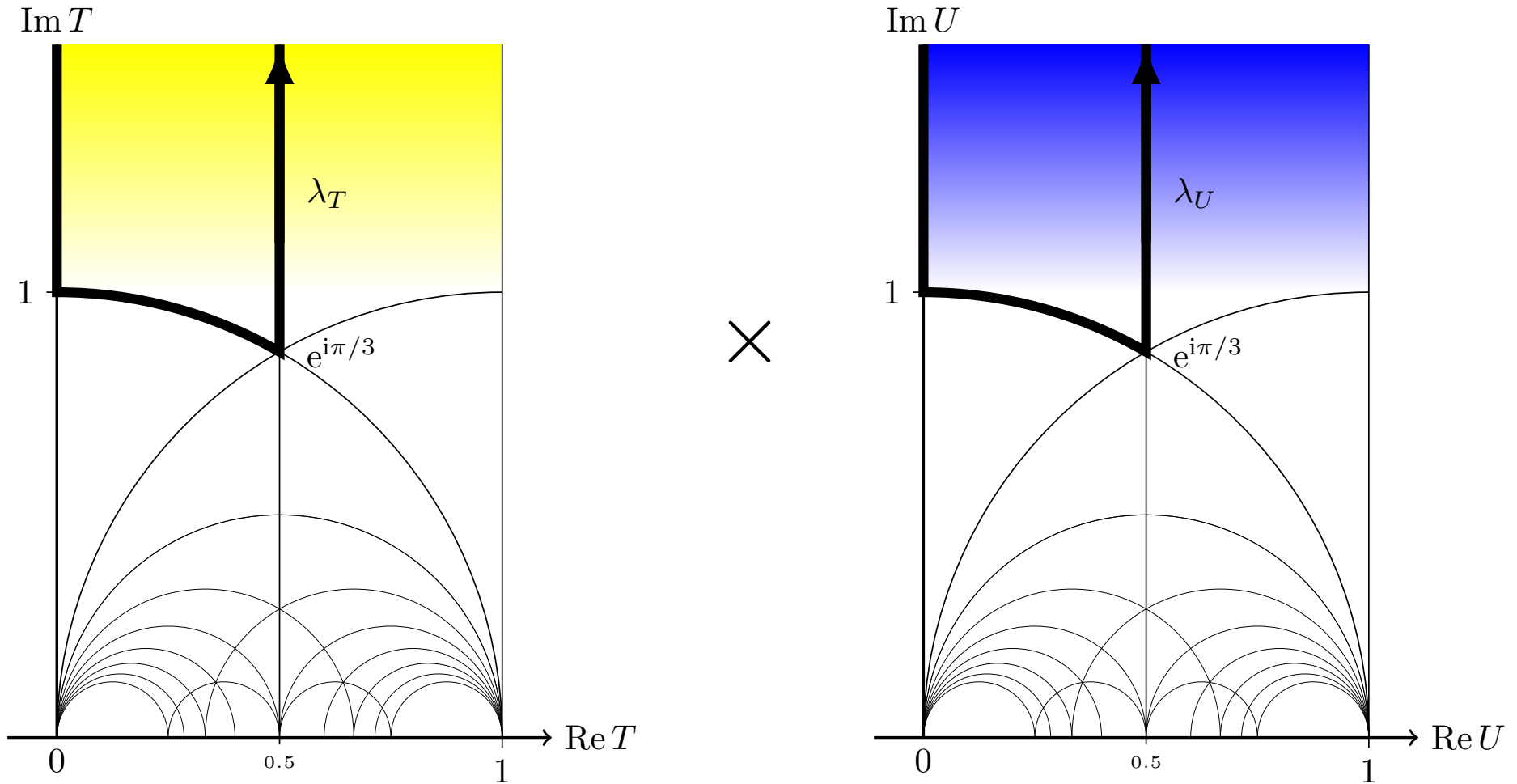


$$S : M \rightarrow -\frac{1}{M}, \quad T : M \rightarrow M + 1 \quad \text{and} \quad U : M \rightarrow -\overline{M}$$

# Moduli space of flavour groups

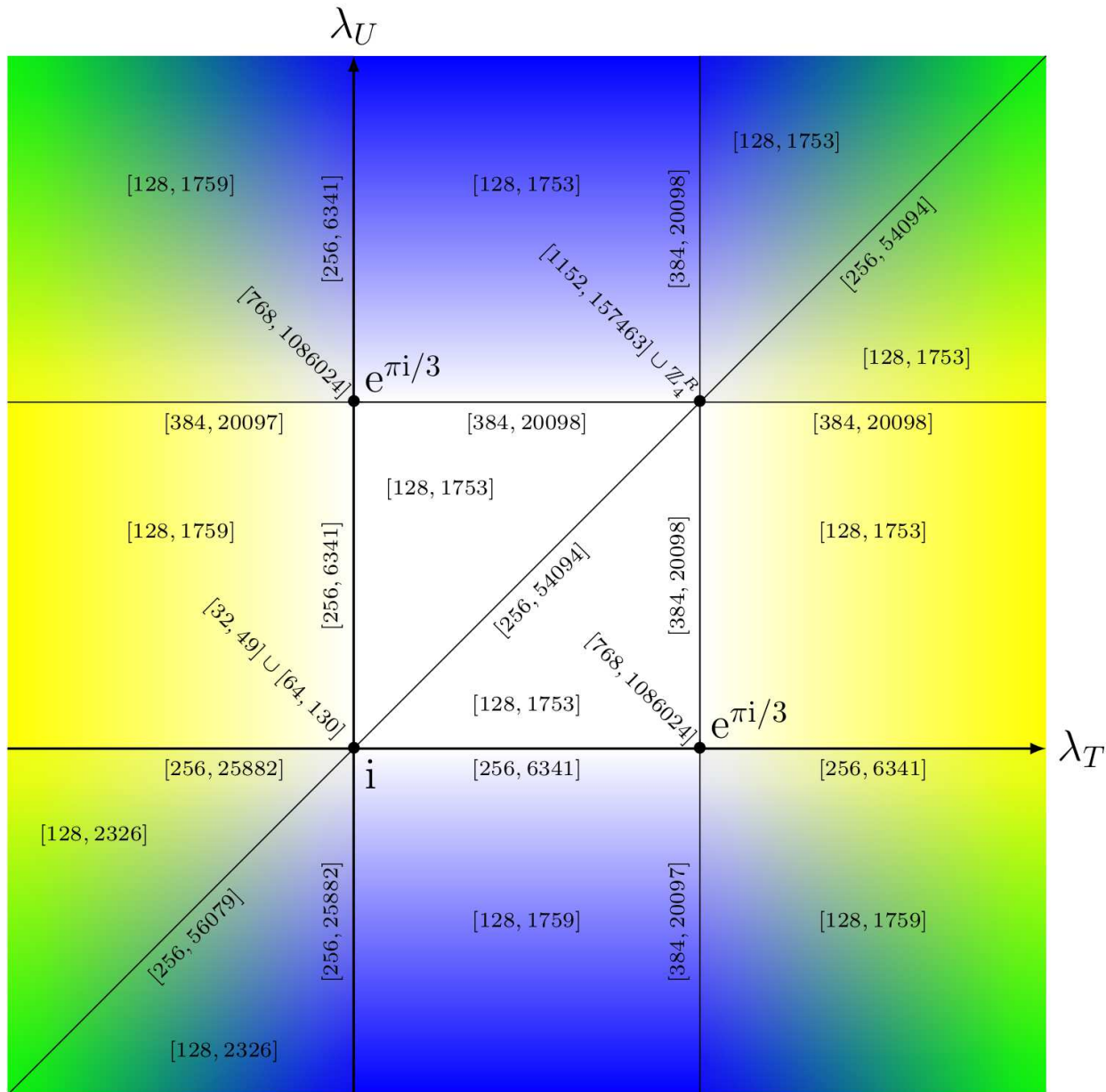


# $Z_2$ -orbifold



Here we have **two** unconstrained moduli:  $T$  and  $U$

# Enhancement



# Messages

The top-down approach to flavor symmetries leads to a

- unification of traditional (discrete) flavor, CP and modular symmetries within an eclectic flavor scheme
- modular flavor symmetry is a prediction of string theory
- traditional flavor symmetry is universal in moduli space
- there are non-universal enhancements (including CP at some places (broken in generic moduli space))
- CP is a natural consequence of the symmetries of the underlying string theory
- the potential flavor groups are large and non-universal
- spontaneous breakdown as motion in moduli space

# Consequences

This opens up a new arena for flavor model building and connections to bottom-up constructions

- We need more explicit string constructions

(Carballo-Perez, Peinado, Ramos-Sanchez, 2018)

- top-down predicts modular symmetries for flavor

- the concept of local flavor symmetries allows different flavor groups for different sectors of the theory, as e.g. quarks and leptons

(Baur, Nilles, Trautner, Vaudrevange, 2019)

- but it is not only the groups but also the representations of matter fields that are relevant. Not all of the possible representations appear in the massless sector.

There is still a huge gap between "top-down and bottom-up"  
Strong restrictions from modular symmetries and weights.

# Summary

String theory provides the necessary ingredients for flavor

- traditional flavor group
- discrete modular flavor group
- a natural candidate for CP
- automorphy factors (modular weights) lead to potential additional ( $R$ -)symmetries

The eclectic flavor group provides the basis:

- this includes a non-universality of flavor symmetry in moduli space
- allows a different flavor structure for quarks and leptons