Top-Down Aspects of Modular and Eclectic Flavor Symmetries

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Outline

- The flavor structure of the Standard Model
- Traditional flavor symmetries
- Modular flavor symmetries
- Eclectic Flavor Group
- Local Flavor Unification
- Origin of hierarchies for masses and mixing angles
- Classification and
  lessons from top-down model building
- Open questions

Importance of localized structures in extra dimensions

(Work with Baur, Knapp-Perez, Liu, Ratz, Ramos-Sanchez, Trautner, Vaudrevange, 2019-23)

The flavor structure of SM

Most of the parameters of the SM concern the flavor sector

- Quark sector: 6 masses, 3 angles and one phase
- Lepton sector: 6 masses, 3 angles, one phase and additional parameters from Majorana neutrino masses

The pattern of parameters

- Quarks: hierarchical masses und small mixing angles
- Leptons: two large and one small mixing angle, hierarchical mass pattern and extremely small neutrino masses

The Flavor structure of quarks and leptons is very different!

Bottom-up approach

Discrete symmetries have been used extensively in the discussion of flavor structures

- Many fits from bottom-up perspective with discrete symmetries ($S_3, A_4, S_4, A_5, \Delta(27), \Delta(54)$ etc.)
- Flavor symmetries seem to require different models for quark and lepton sector (small mixing angles for quarks versus large mixing in lepton sector)
- Flavor symmetries are spontaneously broken. This requires the introduction of so-called flavon fields and additional parameters
- bottom-up model building leads to many reasonable fits for various choices of groups and representations

But we are still missing a top-down explanation of flavor

Traditional vs Modular Symmetries

So far the flavor symmetries had specific properties and we refer to them as traditional flavor symmetries

- they are linearly realised
- need flavon fields for symmetry breakdown

Another type of flavor symmetries are modular symmetries

- motivated by string theory dualities (Lauer, Mas, Nilles, 1989)
- applied recently to lepton sector (Feruglio, 2017)
- nonlinearly realised (no flavon fields needed)
- Yukawa couplings are modular forms

Combine with traditional flavor symmetries to the so-called "eclectic flavor group" (Nilles, Ramos-Sanchez, Vaudrevange, 2020)

String Geometry of extra dimensions

Strings are extended objects and this reflects itself in special aspects of geometry (incl. winding modes).
We have:

- normal symmetries of extra dimensions as observed in quantum field theory – traditional flavor symmetries.
- String duality transformations lead to modular or symplectic flavor symmetries
- They combine to a unified picture within the concept of eclectic flavor symmetries

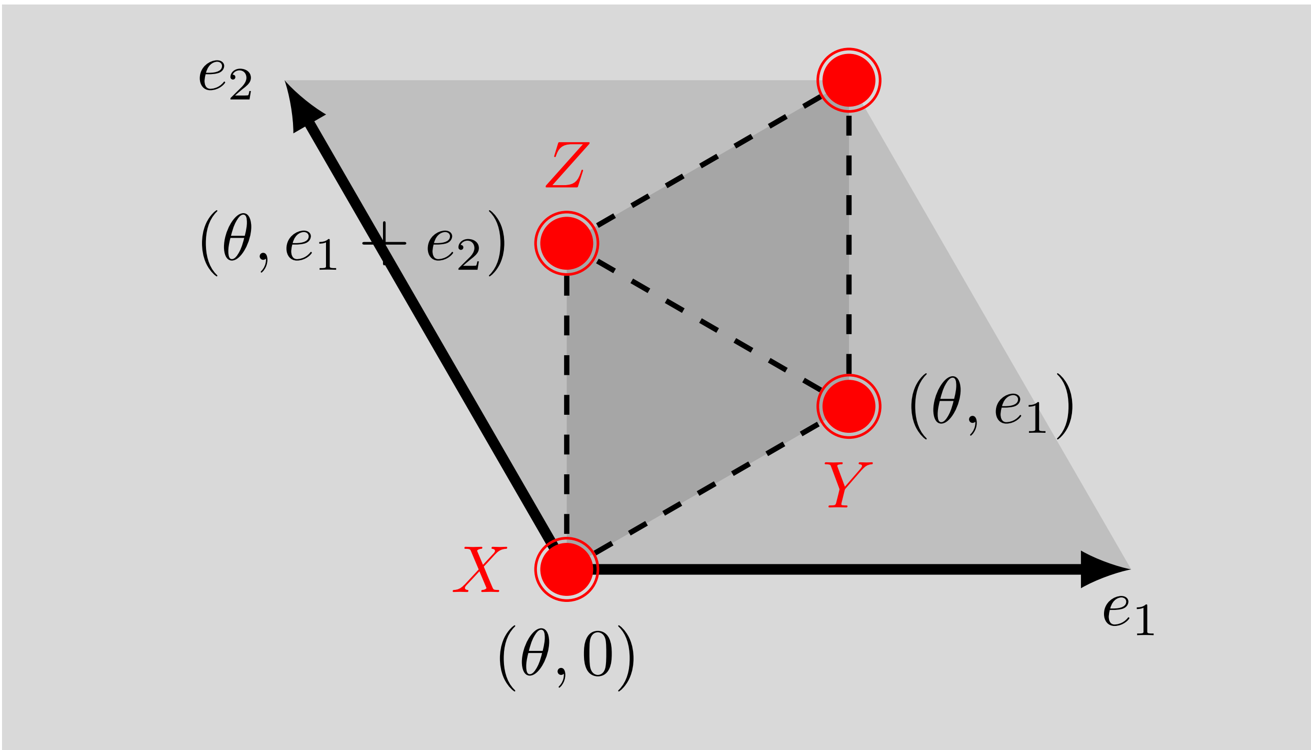
In the following we illustrate with a simple example

- twisted 2D-torus with localized matter fields
- relevant for compactifications with elliptic fibrations

Traditional Flavor Symmetries

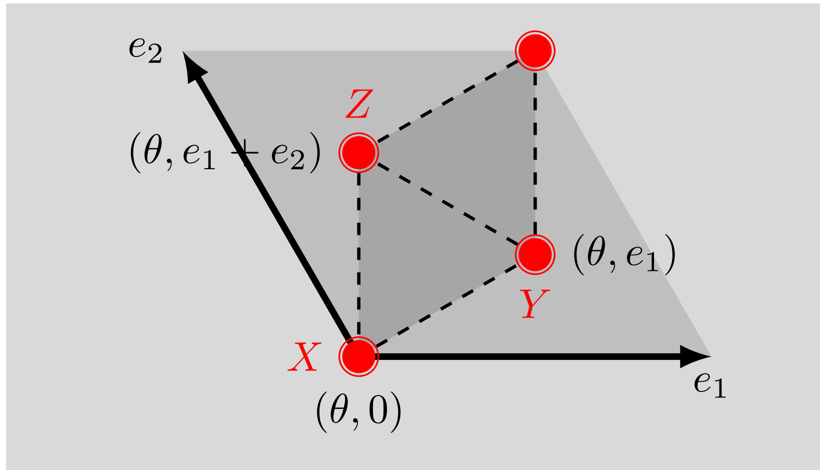
In string theory discrete symmetries can arise form geometry and string selection rules.

As an example we consider the orbifold T_2/Z_3



Discrete symmetry $\Delta(54)$

- untwisted and twisted fields
- S_3 symmetry from interchange of fixed points
- $Z_3 \times Z_3$ symmetry from string theory selection rules



- $\Delta(54)$ as multiplicative closure of S_3 and $Z_3 \times Z_3$
- $\Delta(54)$ – a non-abelian subgroup of $SU(3)_{\text{flavor}}$
- e.g. flavor symmetry for three families of quarks (as triplets of $\Delta(54)$) (Kobayashi, Nilles, Ploger, Raby, Ratz, 2006)

String dualities

Consider a particle on a circle with radius R

- discrete spectrum of momentum modes (KK-modes)
- density of spectrum is governed by m/R (m integer)
- heavy modes decouple for $R \to 0$

Now consider a string

- KK modes as before m/R
- Strings can wind around circle
- spectrum of winding modes governed by nR
- massless modes for $R \to 0$

T-duality

This interplay of momentum and winding modes is the origin of T-duality where one simultaneously interchanges

- momentum $\rightarrow$ winding
- $R \rightarrow 1/R$

This transformation maps a theory to its T-dual theory.

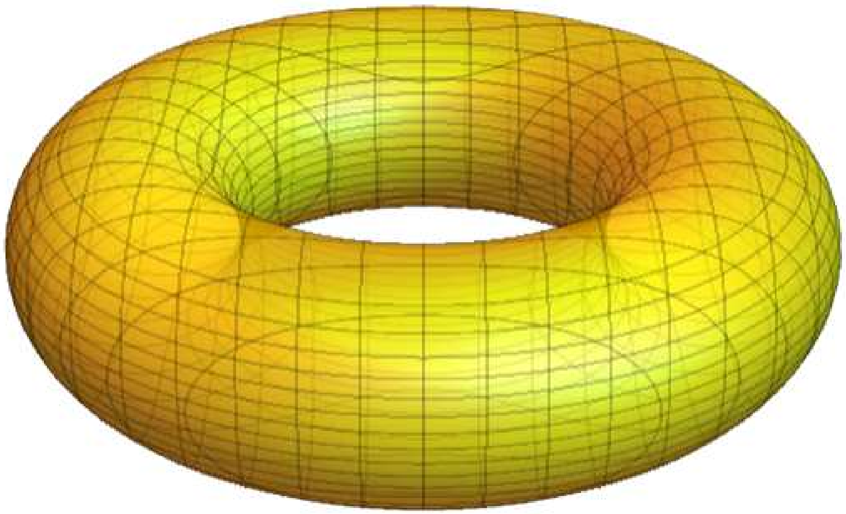
- self-dual point is $R^2 = \alpha' = 1/M^2_{\text{string}}$

If the string scale M_{string} is large, the low energy effective theory describes the momentum states and the winding states are heavy.

Can T-duality play a relevant role for flavor symmetry?

Torus compactification

Strings can wind around several cycles



Complex modulus M (in complex upper half plane)

Modular Transformations

Modular transformations (dualities) exchange windings and momenta and act nontrivially on the moduli of the torus.

In $D = 2$ these transformations are connected to the group $SL(2, Z)$ acting on Kähler and complex structure moduli.

The group $SL(2, Z)$ is generated by two elements

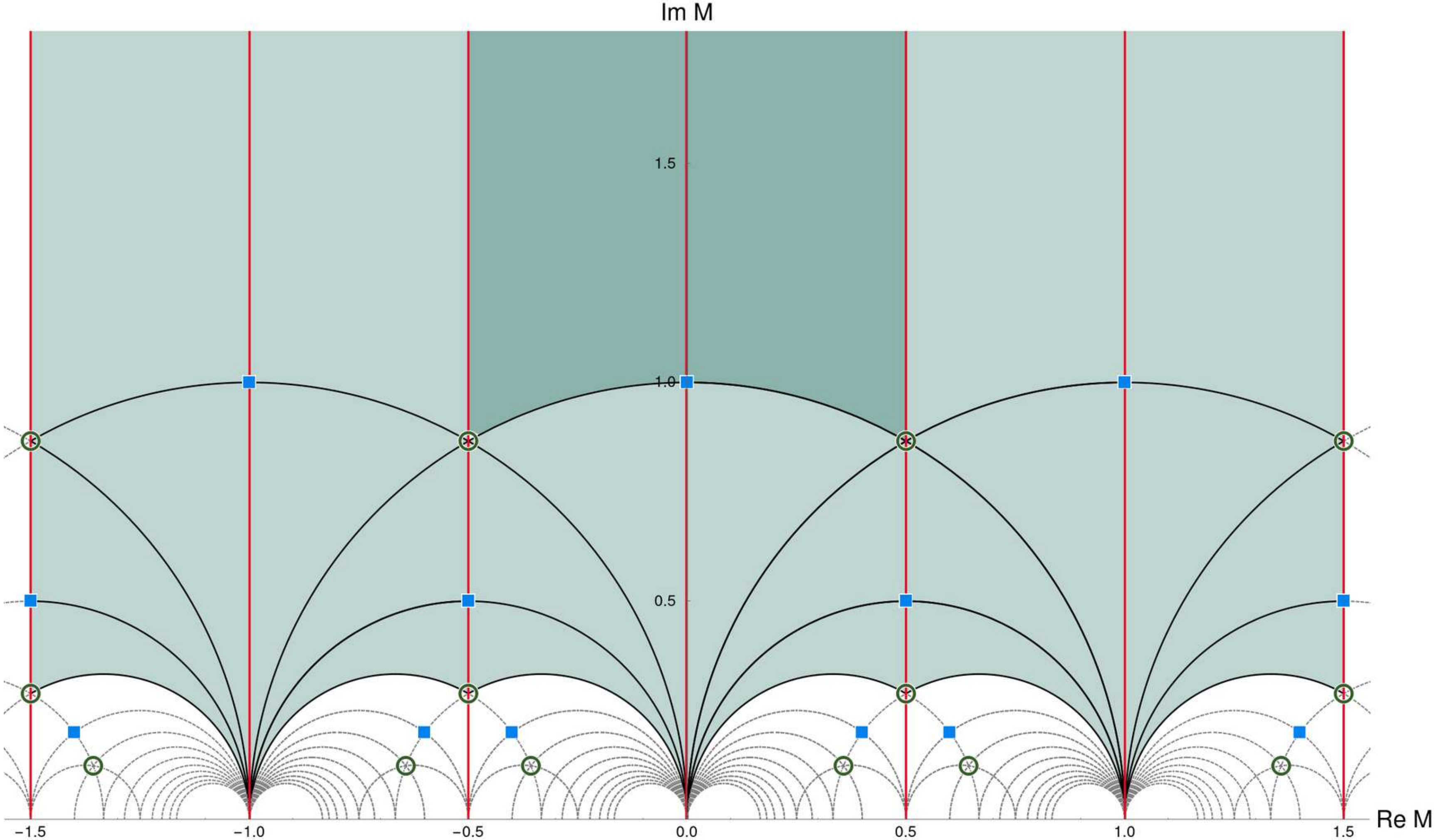
$$\mathrm{S}\ ,\ \mathrm{T}: \quad \text{with}\ \ \mathrm{S}^4 = 1\ \ \text{and}\ \ \mathrm{S}^2 = (\mathrm{ST})^3$$

A modulus M transforms as

$$\mathrm{S}: \quad M \to -\frac{1}{M} \quad \text{and} \quad \mathrm{T}: M \to M + 1$$

Further transformations might include $M \to -\overline{M}$ and mirror symmetry between Kähler and complex structure moduli.

Fundamental Domain



Three fixed points at M$= i$, $\omega = \exp(2\pi i/3)$ and $i\infty$

Modular Forms

String dualities give important constraints on the action of the theory via the modular group $SL(2,Z)$:

$$\gamma: \quad M \to \frac{aM+b}{cM+d}$$

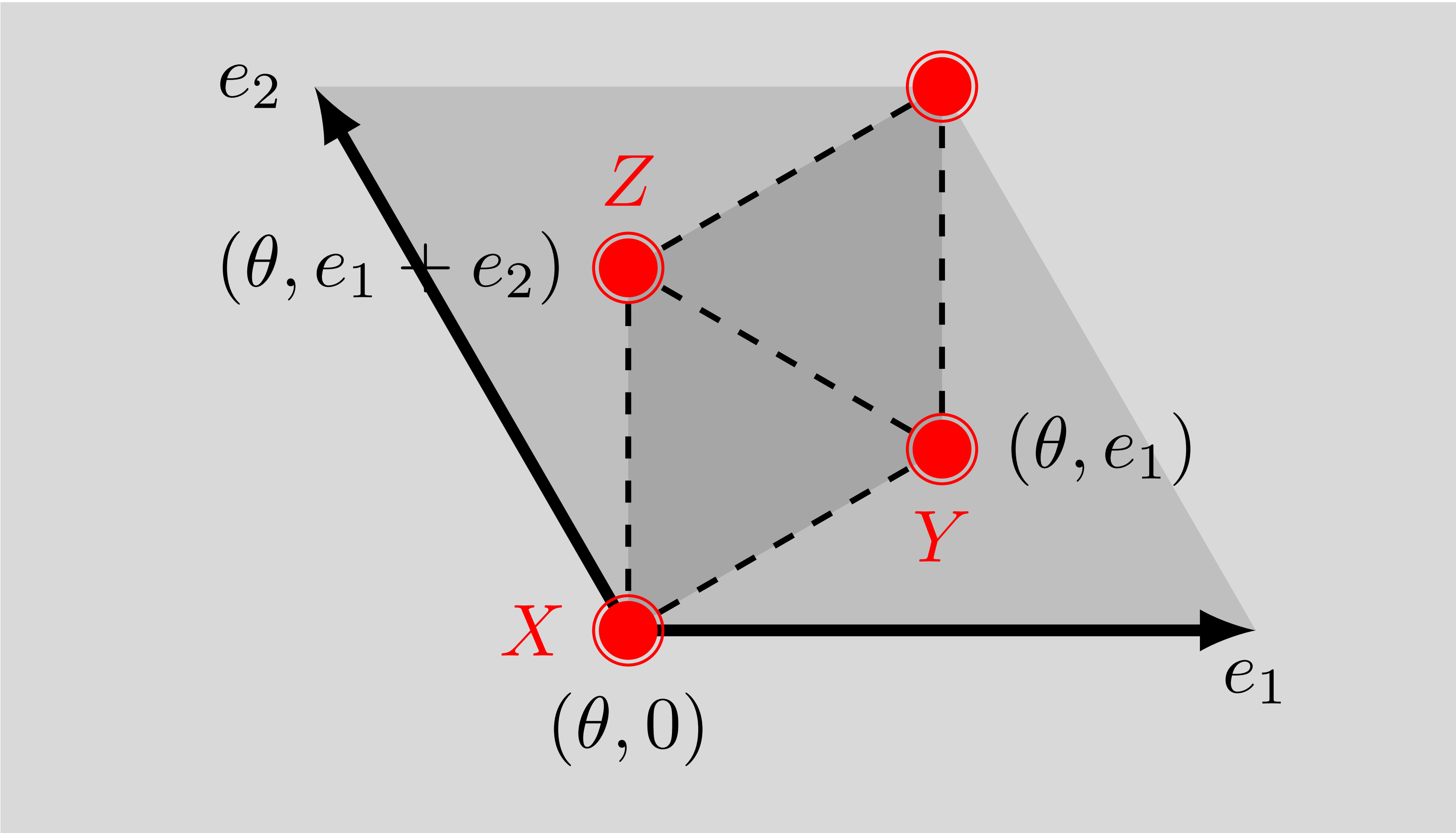
with $ad-bc=1$ and integer a,b,c,d.

Matter fields transform as representations $\rho(\gamma)$ and modular functions of weight k

$$\gamma: \quad \phi \to (cM+d)^k \rho(\gamma)\phi \,.$$

Yukawa-couplings transform as modular functions as well.
$G = K + \log|W|^2$ must be invariant under T-duality

Towards Modular Flavor Symmetry



Modular flavor symmetry

On the T_2/Z_3 orbifold some of the moduli are frozen,

- lattice vectors e_1 and e_2 have the same length
- angle is 120 degrees

Modular transformations form a subgroup of $SL(2, Z)$

- $\Gamma(3) = SL(2, 3Z)$ as a mod(3) subgroup of $SL(2, Z)$
- discrete modular flavor group $\Gamma'_3 = SL(2, Z)/\Gamma(3)$
- the discrete modular group is $\Gamma'_3 = T' \sim SL(2, 3)$ (which acts nontrivially on twisted fields); the double cover of $\Gamma_3 \sim A_4$ (which acts only on the modulus).
- the CP transformation $M \to -\overline{M}$ completes the picture.

Full discrete modular group is $GL(2, 3)$.

Eclectic Flavor Groups

We have thus two types of flavor groups

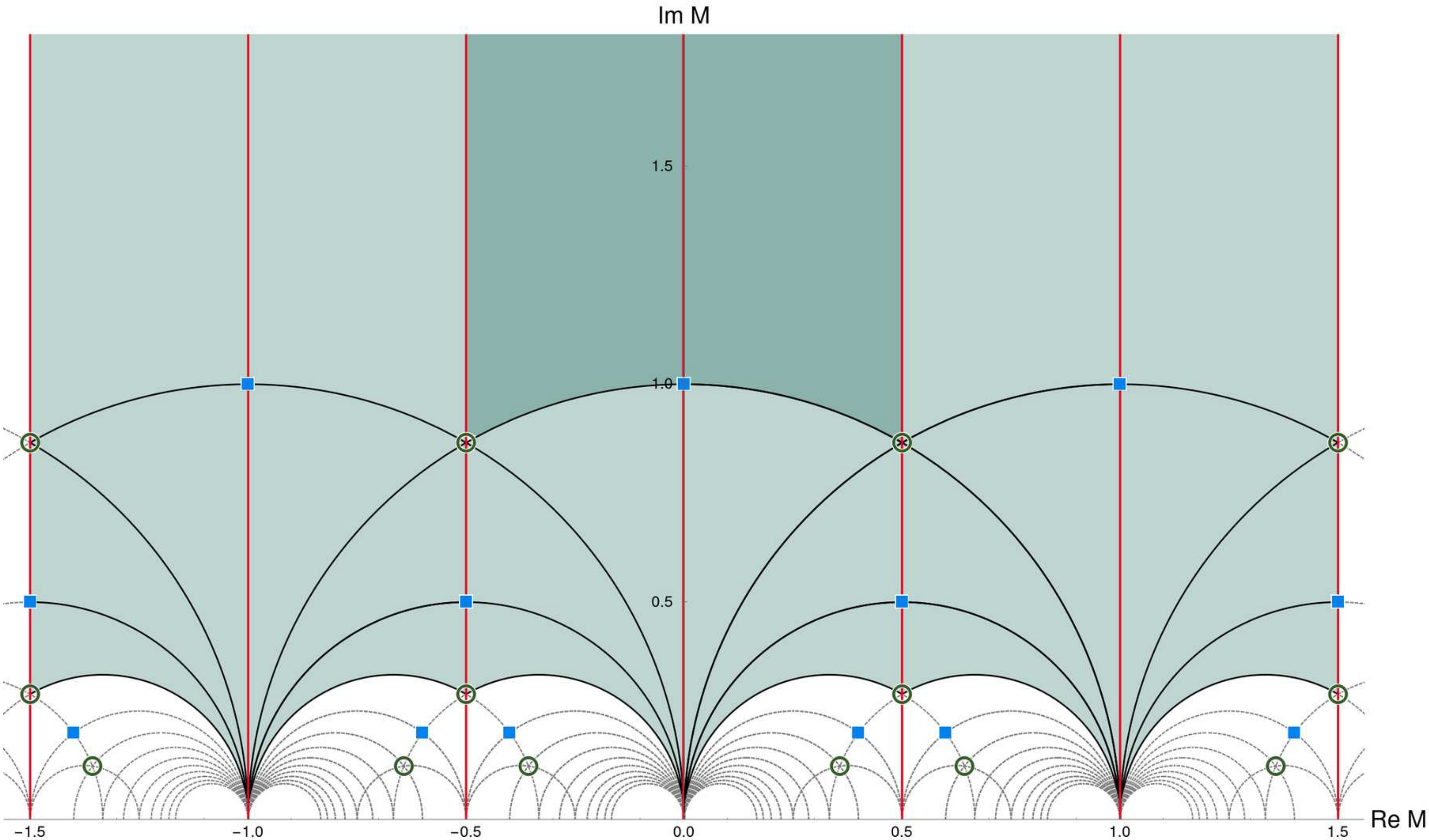
- the traditional flavor group that is universal in moduli space (here $\Delta(54)$)
- the modular flavor group that transforms the moduli nontrivially (here T')

The eclectic flavor group is defined as the multiplicative closure of these groups. Here we obtain for T_2/Z_3

- $\Omega(1) = SG[648, 533]$ from $\Delta(54)$ and $T' = SL(2, 3)$
- $SG[1296, 2891]$ from $\Delta(54)$ and $GL(2, 3)$ including CP

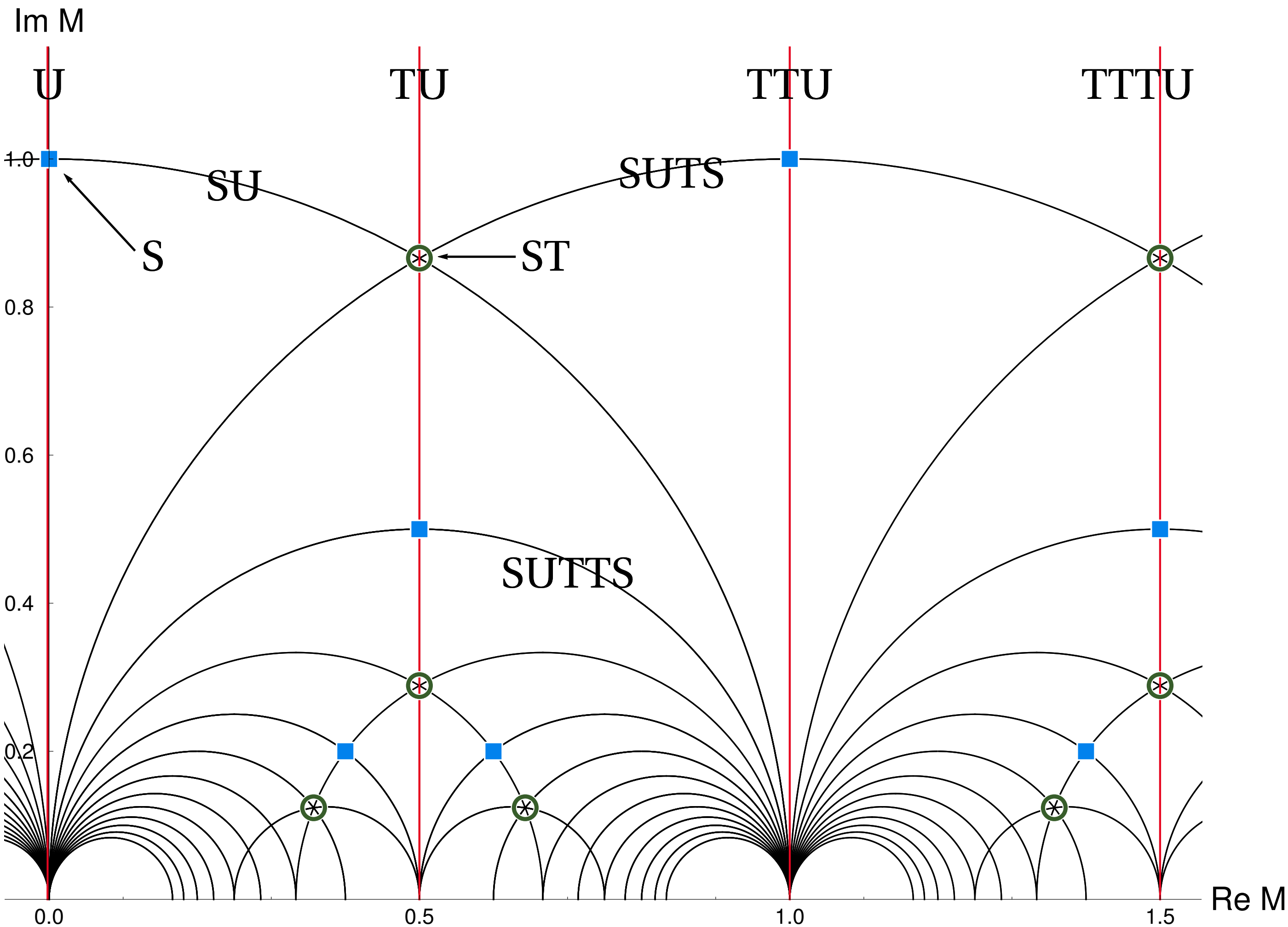
The eclectic group is the largest possible flavor group for the given system, but it is not necessarily linearly realized.

Local Flavor Unification



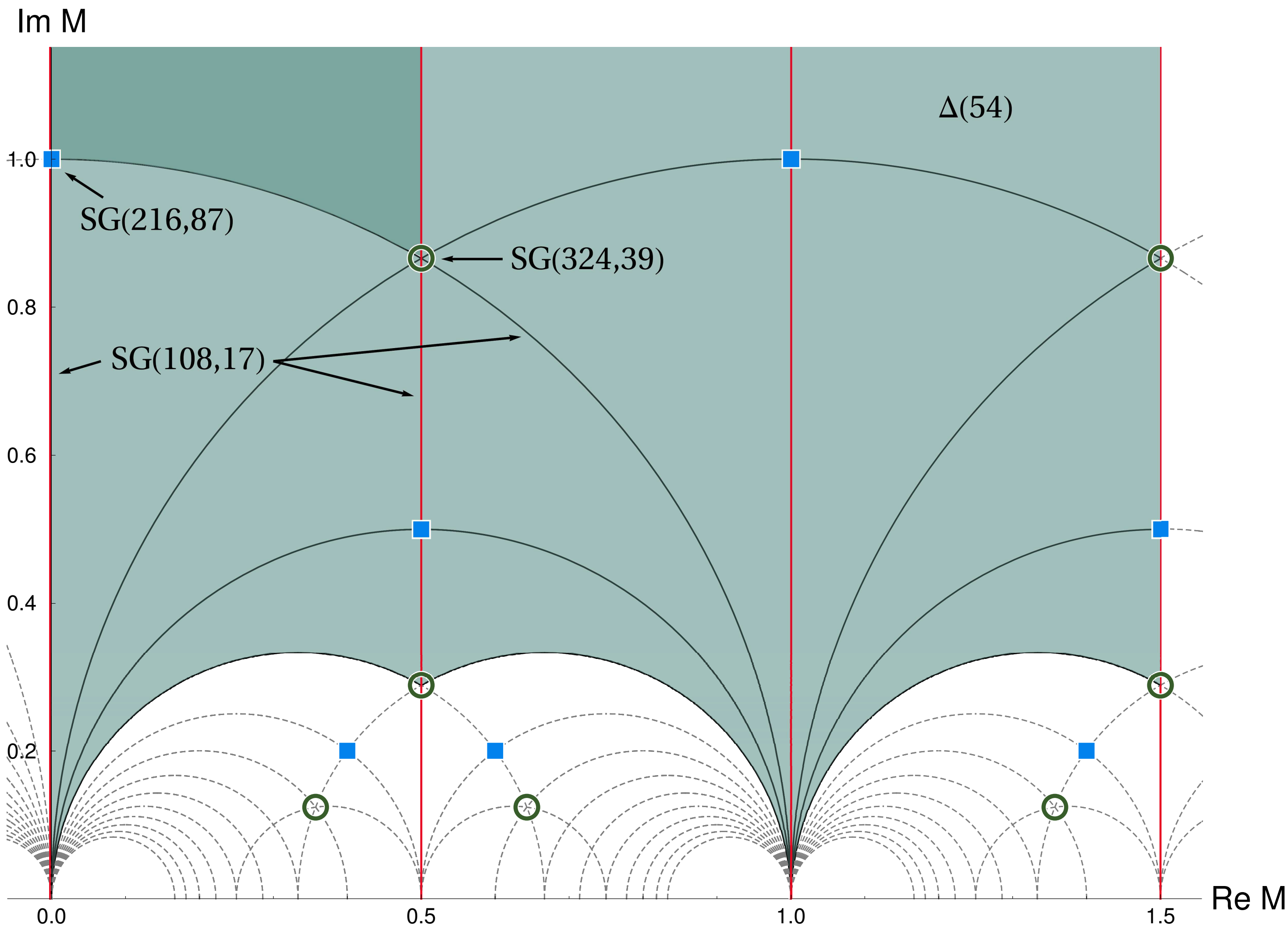
Moduli space of $\Gamma(3)$

Fixed lines and points



$$\mathrm{S}: M \to -\frac{1}{M}, \quad \mathrm{T}: M \to M+1 \quad \text{and} \quad \mathrm{U}: M \to -\overline{M}$$

Moduli space of flavour groups



"Local Flavor Unification"

Unification of Flavor and CP

Summary of predictions of the string picture:

- traditional flavor symmetries (universal in moduli space)
- modular flavor symmetries and CP are non-universal in moduli space

They unify in the eclectic picture of flavor symmetry.
You cannot just have one without the other.

The non-universality in moduli space leads to

- local flavor unification at specific points in moduli space
- hierarchical structures of masses, mixing angles and phases in vicinity of fixed points or lines
- potentially different pictures for quarks and leptons

Classification

- Modular symmetry $SL(2, Z)$ is intrinsically related to the 2-torus T^2 with two moduli T and U
- Chiral fermions require a twist Z_k of T^2, embedded in 6-dimensiponal compact space.
- relevant Z_k are $k = 2, 3, 4, 6$
- for $k = 3, 4, 6$ the T-modulus is fixed to allow for the twist
- higher k lead to larger modular symmetries Γ_k
- at the expense of smaller traditional flavor symmetries
- $k = 3$: $\Omega(1) = SG[648, 533]$ from $\Delta(54)$ and $T' = SL(2, 3)$
- embedding in 6-dimensional space gives additional R-symmetries

Z_4 and Z_6

The 2-dimensional Z_3 orbifold gives already a promising result. What about the others?

- Z_6 gives modular group $\Gamma_2 \times \Gamma_3' = S_3 \times T' = [144, 128]$
- traditional flavor group is abelian (one fixed point)
- Z_6 eclectic group is $[144, 128] \times Z_{36}^R$ (5184 elements)
- Z_4 gives S_4' modular group with an intrinsic relation to the traditional flavor and R-symmetry
- traditional flavor symmetry is $[64, 185]$ (including the R-symmetry) and Z_4 eclectic group is $[384, 5614]$

Interpretation of quark- and lepton-multipets is less clear than in the Z_3 case

(Work in progress)

Z_2 orbifold: two moduli

Here the twist does not constrain the moduli T and U

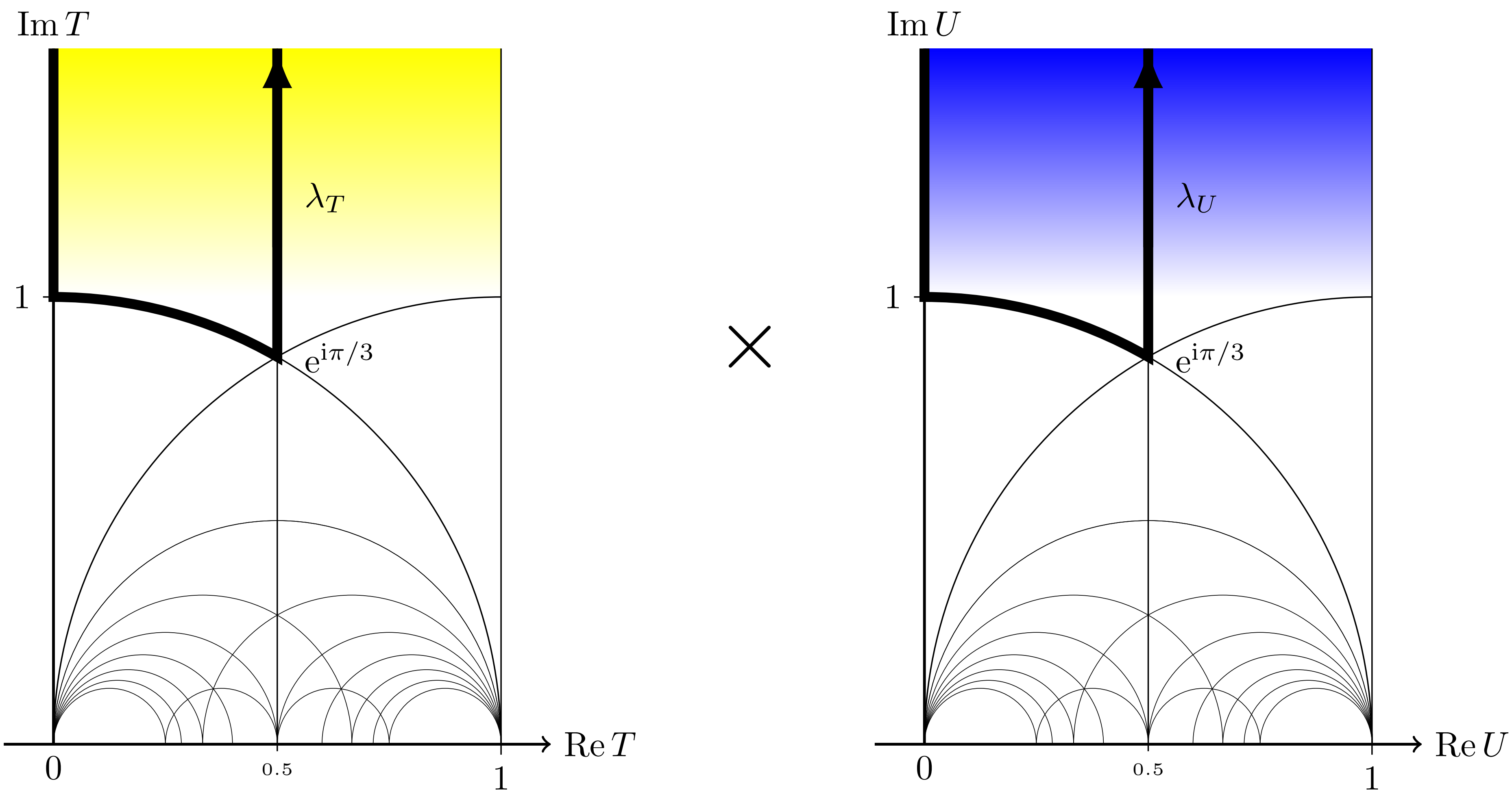
- and we have the full $SL(2,Z)_T \times SL(2,Z)_U$.
- The discrete modular group is $\Gamma_2 \times \Gamma_2 \times Z_2$,
- where $\Gamma_2 = S_3$ and
- Z_2 interchanges T and U (known as mirror symmetry).
- The traditional flavor group is the product of $(D_8 \times D_8)/Z_2$ and a Z_4 R-symmetry.

This leads to an

- eclectic group with 2304 elements (excluding CP)
- or 4608 elements (including CP)

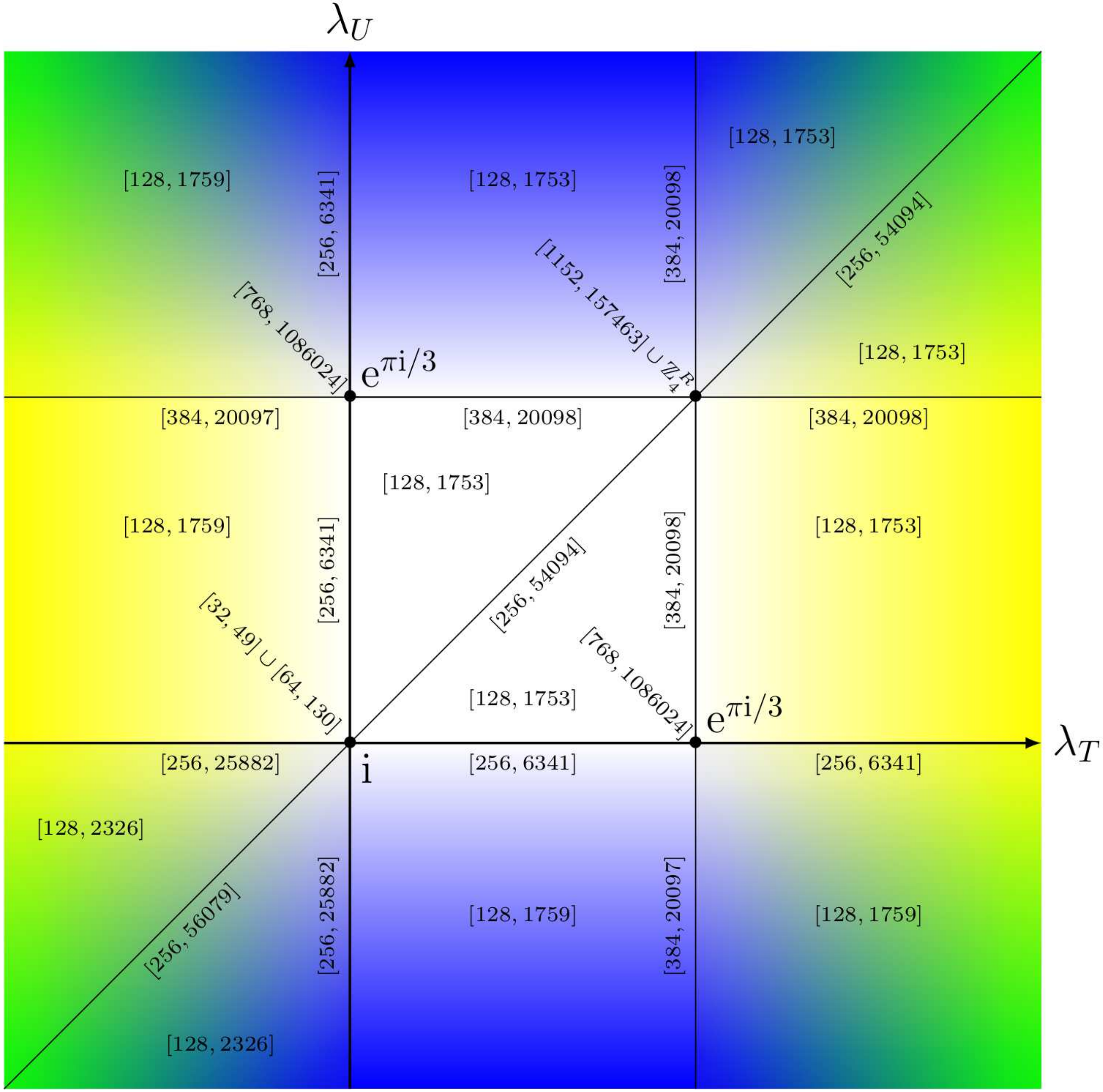
with a rich pattern of local flavor group enhancements.

Z_2-orbifold

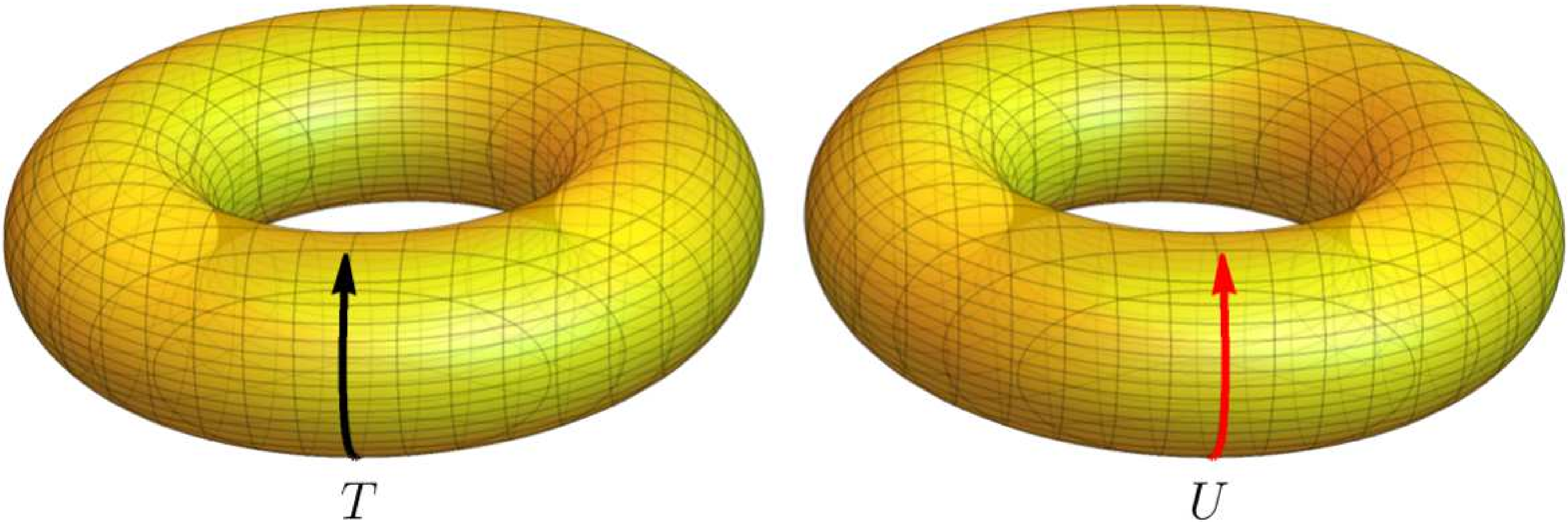


Here we have two unconstrained moduli: T and U

Enhancement

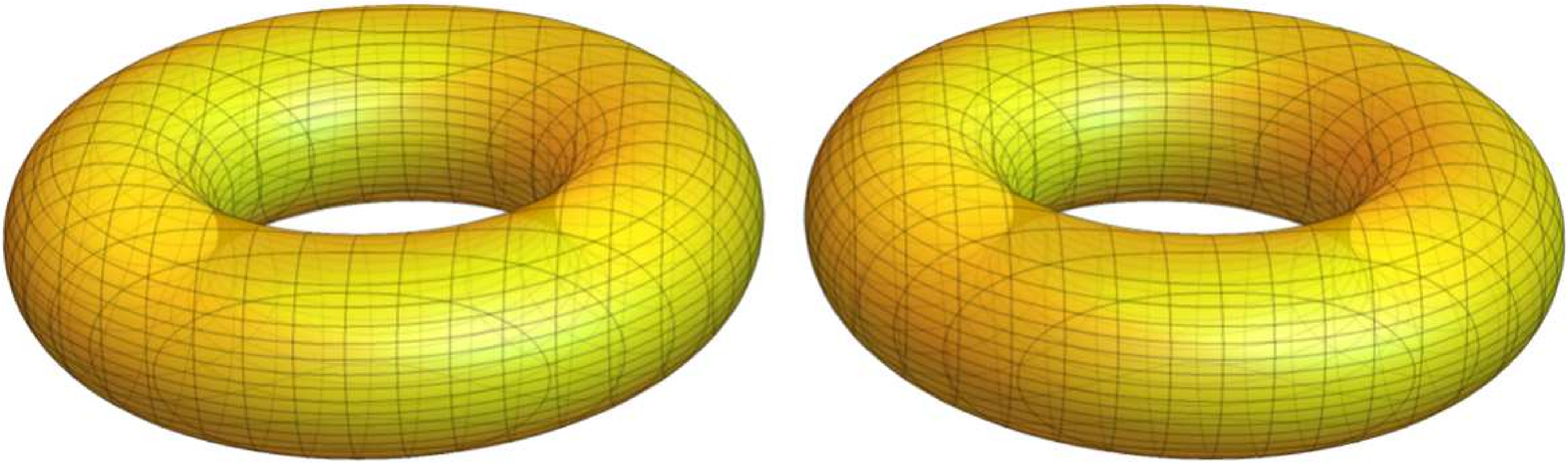


Auxiliary Surface: Double Torus



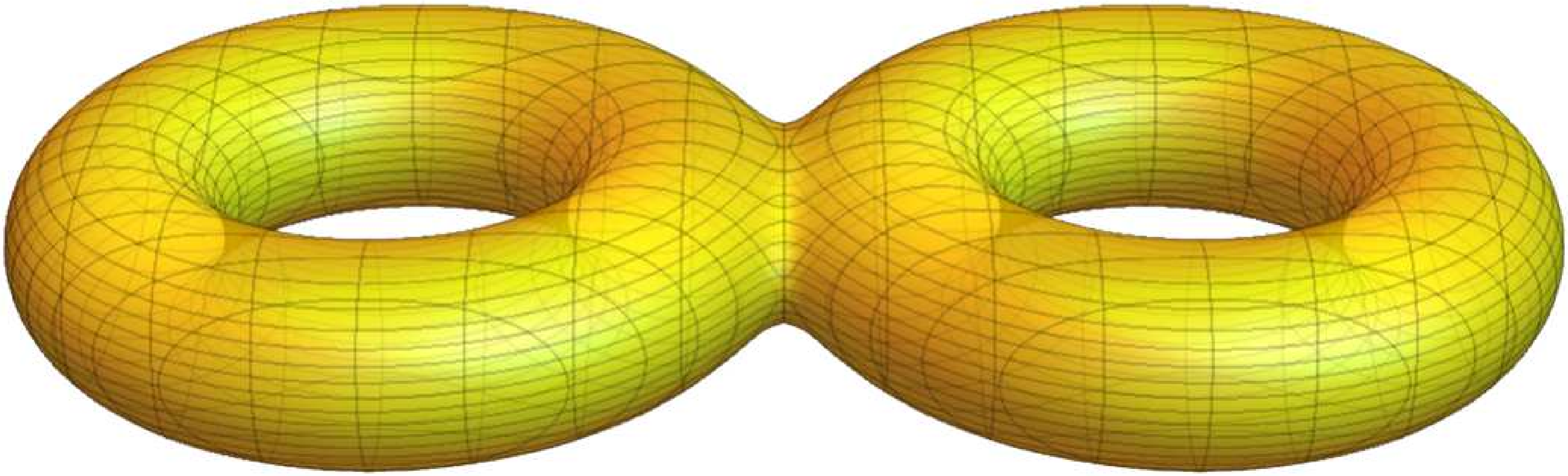
Auxiliary surface for two moduli: $SL(2,Z)_T \times SL(2,Z)_U$

Riemann surface of genus 2



Auxiliary surface for two moduli: T and U

Riemann Surface of Genus 2



Auxiliary surface with three moduli: T + U + Wilson line

Siegel Modular Forms

This leads to a generalization of the modular group to larger groups $Sp(2g, Z)$ characterized through Riemann surfaces of higher genus g: (Ding, Feruglio, Liu, 2021)

- for $g = 2$ the Siegel modular group $Sp(4, Z)$
- includes $SL(2, Z)_{U,T}$ and describes three moduli.
- Fundamental domain (6 points, 5 lines, 2 surfaces)
- Orbifold twists are connected to fixed loci in fundamental domain
- Discrete modular group $\Gamma_{g,k}$ ($\Gamma_{1,k} = \Gamma_k$)
- $\Gamma_{2,2} = S_6$ includes $S_3 \times S_3$ and mirror symmetry
- $\Gamma_{2,3}$ has already 51840 elements (work in progress)

Messages

The top-down approach to flavor symmetries leads to a

- unification of traditional (discrete) flavor, CP and modular symmetries within an eclectic flavor scheme
- modular flavor symmetry is a prediction of string theory
- traditional flavor symmetry is universal in moduli space
- there are non-universal enhancements (including CP at some places (broken in generic moduli space))
- CP is a consequence of the underlying string theory
- spontaneous breakdown as motion in moduli space
- nonlinearly realized symmetries allow for uncontrollable Kähler corrections (Chen, Ramos-Sanchez, Ratz, 2020; Chen, Knapp-Perez, Ramos-Hamud, Ramos-Sanchez, Ratz, 2022)

Top-Down versus Bottom-Up

This opens up a new arena for flavor model building:

- so far $\Delta(54) \times T'$ is the favourite "top-down" model
- need more explicit string constructions
- but it is not only the groups but also the representations and modular weights of matter fields that are relevant (top-down models very restrictive)
- there is still a huge gap between "top-down" and "bottom-up" constructions
- modular flavour group from outer automorphisms of traditional flavor group (Nilles, Ramos-Sanchez, Vaudrevange, 2020)
- recently applied in "bottom-up" constructions for $\Delta(27)$ as traditional flavor symmetry (see talk by Xiang-Gan Liu)

Open Questions

So far $\Delta(54) \times T'$ seems to be the favourite model

- numerous bottom-up models with these groups
- successful realistic string model from Z_3 orbifold

(see talk by Saul Ramos-Sanchez)

It has been observed that many of the successful fits are in the vicinity of fixed points and lines

(Feruglio, 2022-23; Petcov, Tanimoto, 2022; Abe, Higaki, Kawamura, Kobayashi, 2023)

- moduli stabilization favours boundary of moduli space, but leads to AdS-minima
- uplift moves them slightly away from the boundary and leads to flavor hierarchies (see talk by Saul Ramos-Sanchez)

Summary

String theory provides the necessary ingredients for flavor:

- traditional flavor group
- discrete modular flavor group
- a natural candidate for CP
- the concept of local flavour unification

The eclectic flavor group provides the basis:

- this includes a non-universality of flavor symmetry in moduli space
- allows a different flavor structure for quarks and leptons