Modular Flavor and CP symmetry from a Top-Down Perspective

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Outline

- The flavor structure of the Standard Model
- Traditional flavor symmetries
- Modular flavor symmetries
- Eclectic Flavor Group
- Local Flavor Unification
- Origin of hierarchies for masses and mixing angles
- Lessons from top-down model building
- Global fit for lepton masses and mixing angles

Importance of localized structures in extra dimensions

(Work with A. Baur, M. Kade, A. Trautner, S. Ramos-Sanchez, P. Vaudrevange, 2019-22)

The flavor structure of SM

Most of the parameters of the SM concern the flavor sector

- Quark sector: 6 masses, 3 angles and one phase
- Lepton sector: 6 masses, 3 angles, one phase and additional parameters from Majorana neutrino masses

The pattern of parameters

- Quarks: hierarchical masses und small mixing angles
- Leptons: two large and one small mixing angle, hierarchical mass pattern and extremely small neutrino masses

The Flavor structure of quarks and leptons is very different!

Bottom-up approach

Discrete symmetries have been used extensively in the discussion of flavor structures

- Many fits from bottom-up perspective with discrete symmetries ($S_3, A_4, S_4, A_5, \Delta(27), \Delta(54)$ etc.)
- Flavor symmetries seem to require different models for quark and lepton sector (small mixing angles for quarks versus large mixing in lepton sector)
- Flavor symmetries are spontaneously broken. This requires the introduction of so-called flavon fields and additional parameters
- bottom-up model building leads to many reasonable fits for various choices of groups and representations

But we are still missing a top-down explanation of flavor

String Geometry of extra dimensions

Strings are extended objects and this reflects itself in special aspects of geometry. We have

- normal symmetries of extra dimensions as observed in quantum field theory – traditional flavor symmetries
- String duality transformations lead to modular or symplectic flavor symmetries
- They combine to a unified picture within the concept of eclectic flavor symmetries

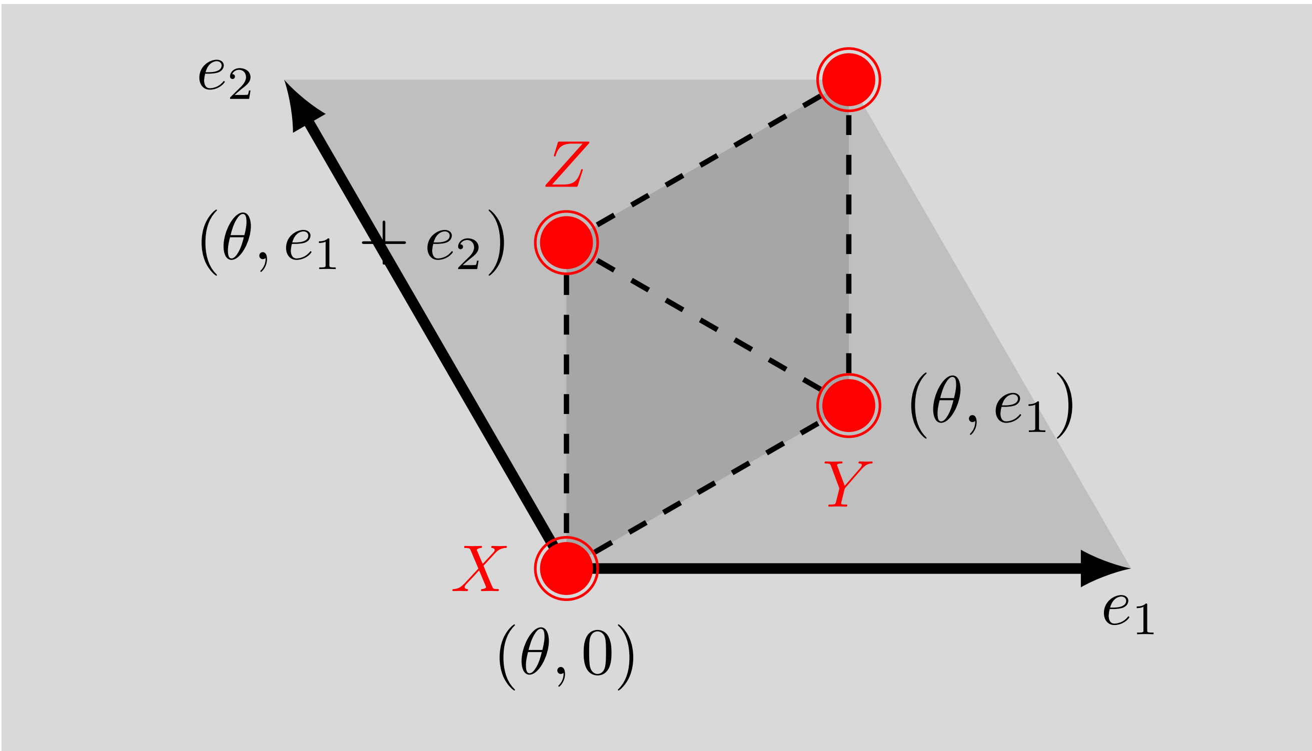
In the following we shall illustrate these symmetries in the case of a simple example

- twisted 2D-torus with localized matter fields
- relevant for compactifications with elliptic fibrations

Traditional Flavor Symmetries

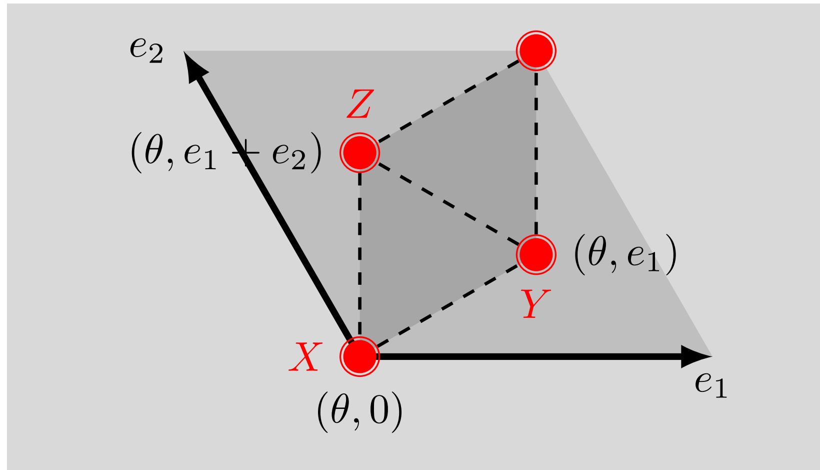
In string theory discrete symmetries can arise form geometry and string selection rules.

As an example we consider the orbifold T_2/Z_3



Discrete symmetry $\Delta(54)$

- untwisted and twisted fields
- S_3 symmetry from interchange of fixed points
- $Z_3 \times Z_3$ symmetry from string theory selection rules



- $\Delta(54)$ as multiplicative closure of S_3 and $Z_3 \times Z_3$
- $\Delta(54)$ – a non-abelian subgroup of $SU(3)_{\text{flavor}}$
- e.g. flavor symmetry for three families of quarks (as triplets of $\Delta(54)$)

String dualities

Consider a particle on a circle with radius R

- discrete spectrum of momentum modes (KK-modes)
- density of spectrum is governed by m/R (m integer)
- heavy modes decouple for $R \to 0$

Now consider a string

- KK modes as before m/R
- Strings can wind around circle
- spectrum of winding modes governed by nR
- massless modes for $R \to 0$

T-duality

This interplay of momentum and winding modes is the origin of T-duality where one simultaneously interchanges

- momentum $\rightarrow$ winding
- $R \rightarrow 1/R$

This transformation maps a theory to its T-dual theory.

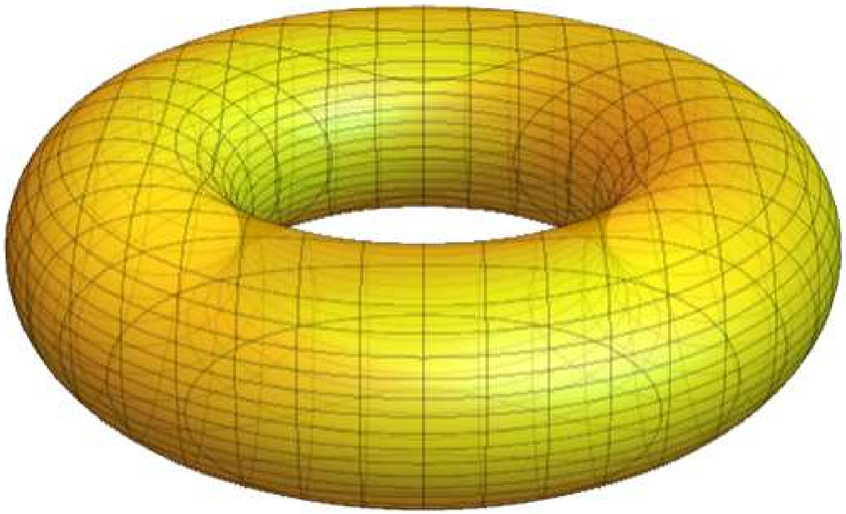
- self-dual point is $R^2 = \alpha' = 1/M_{\text{string}}^2$

If the string scale M_{string} is large, the low energy effective theory describes the momentum states and the winding states are heavy.

Can T-duality play a relevant role for flavor symmetry?

Torus compactification

Strings can wind around several cycles



Complex modulus M (in complex upper half plane)

Modular Transformations

Modular transformations (dualities) exchange windings and momenta and act nontrivially on the moduli of the torus.

In $D = 2$ these transformations are connected to the group $SL(2, Z)$ acting on Kähler and complex structure moduli.

The group $SL(2, Z)$ is generated by two elements

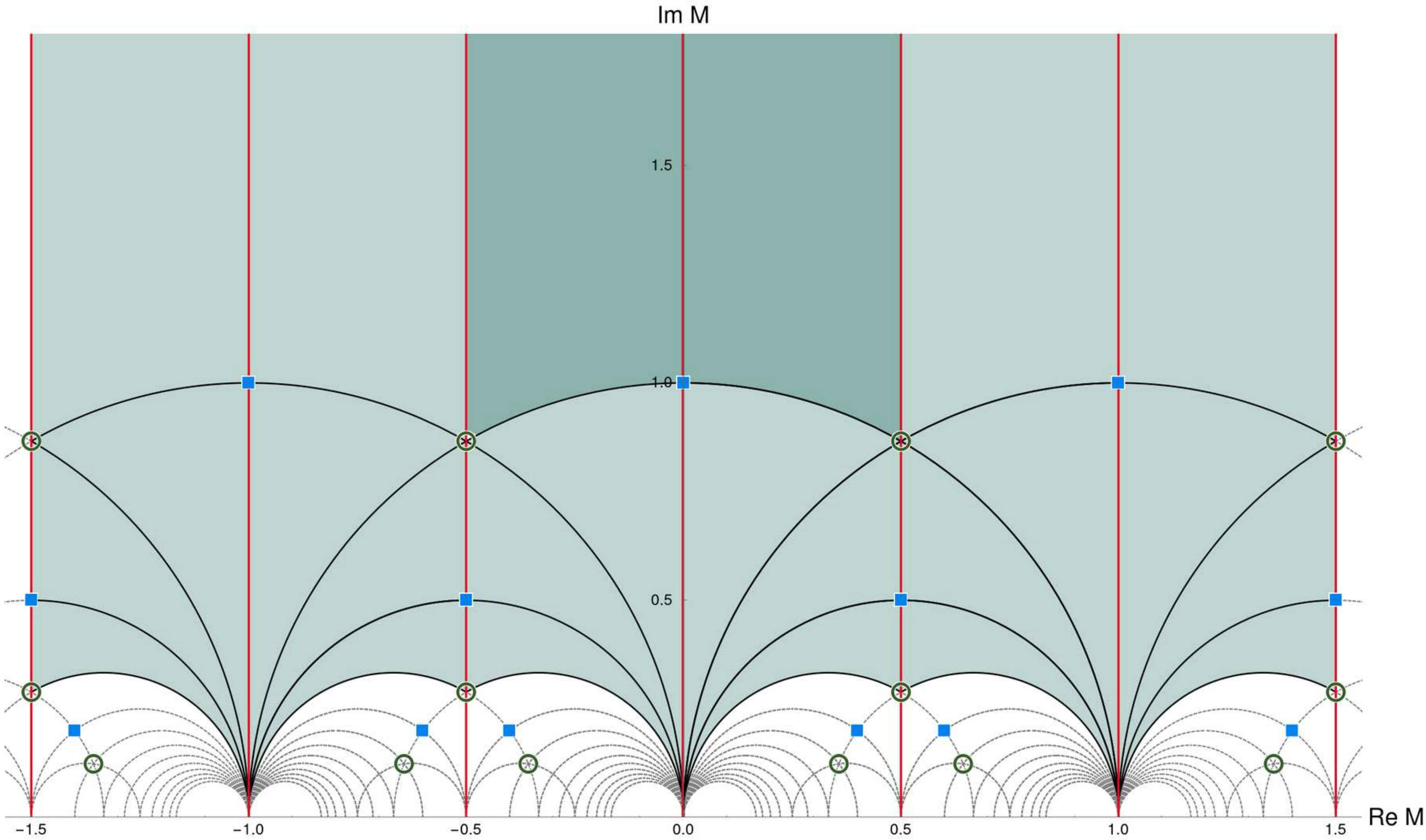
$$\mathrm{S}\ ,\ \mathrm{T}: \quad \text{with}\ \ \mathrm{S}^4 = 1\ \ \text{and}\ \ \mathrm{S}^2 = (\mathrm{ST})^3$$

A modulus M transforms as

$$\mathrm{S}: \quad M \to -\frac{1}{M} \quad \text{and} \quad \mathrm{T}: M \to M + 1$$

Further transformations might include $M \to -\overline{M}$ and mirror symmetry between Kähler and complex structure moduli.

Fundamental Domain



Modular Forms

String dualities give important constraints on the action of the theory via the modular group $SL(2,Z)$:

$$\gamma: \quad M \rightarrow \frac{aM+b}{cM+d}$$

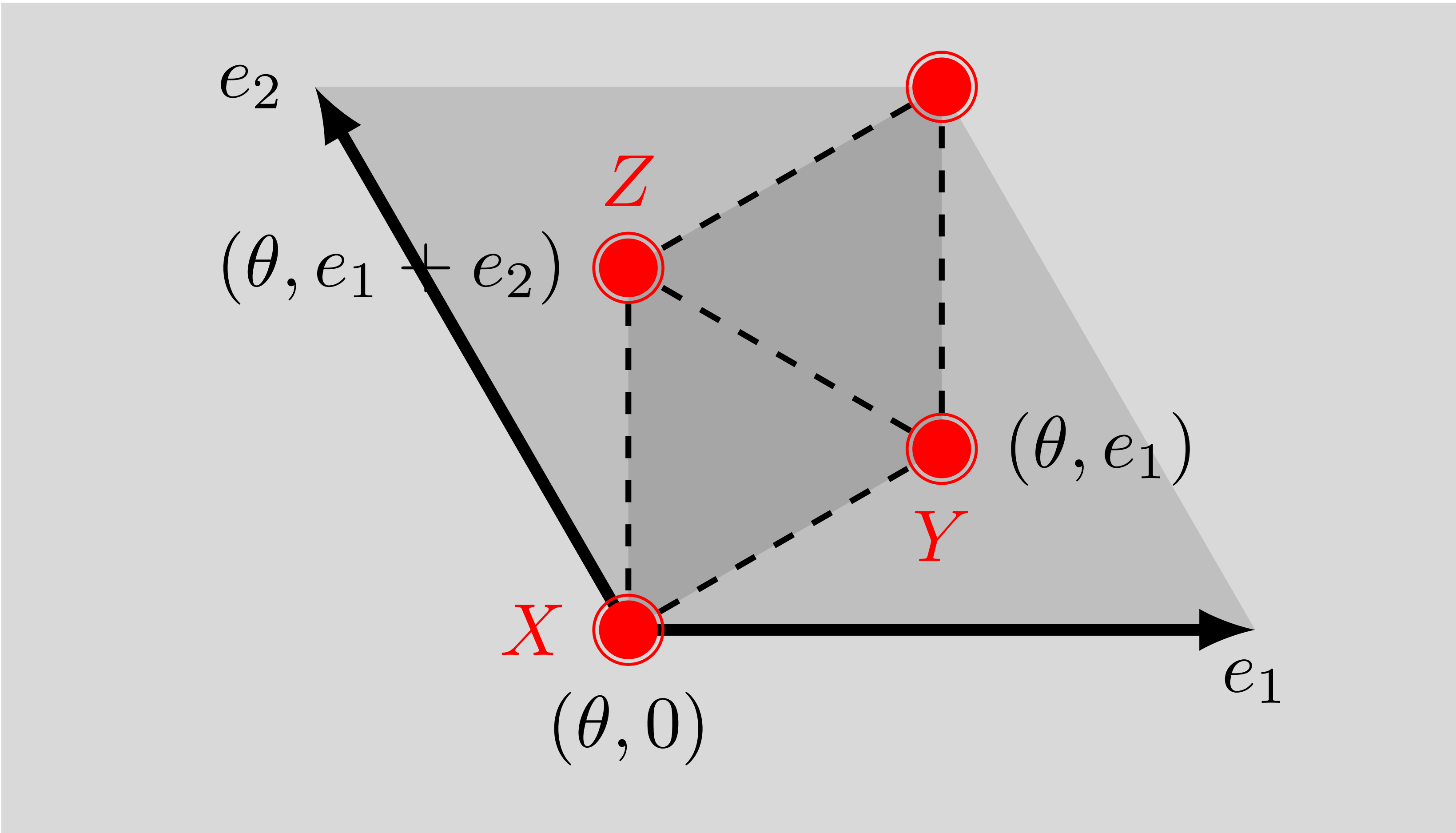
with $ad-bc=1$ and integer a,b,c,d.

Matter fields transform as representations $\rho(\gamma)$ and modular functions of weight k

$$\gamma: \quad \phi \rightarrow (cM+d)^k \rho(\gamma)\phi \, .$$

Yukawa-couplings transform as modular functions as well.
$G = K + \log|W|^2$ must be invariant under T-duality

Orbifold T_2/Z_3



Modular flavor symmetry

On the T_2/Z_3 orbifold some of the moduli are frozen

- lattice vectors e_1 and e_2 have the same length
- angle is 120 degrees

Modular transformations form a subgroup of $SL(2,Z)$

- $\Gamma(3)$ as a mod(3) subgroup of $SL(2,Z)$
- discrete modular flavor group $\Gamma_3 = SL(2,Z)/\Gamma(3)$
- here the full discrete modular group is not just $\Gamma_3 \sim A_4$ but its double cover $T' \sim SL(2,3)$ (which acts nontrivially on the twisted fields)
- the CP transformation $M \to -\overline{M}$ completes the picture.

Full discrete modular group is $GL(2,3)$.

Eclectic Flavor Groups

We have thus two types of flavor groups

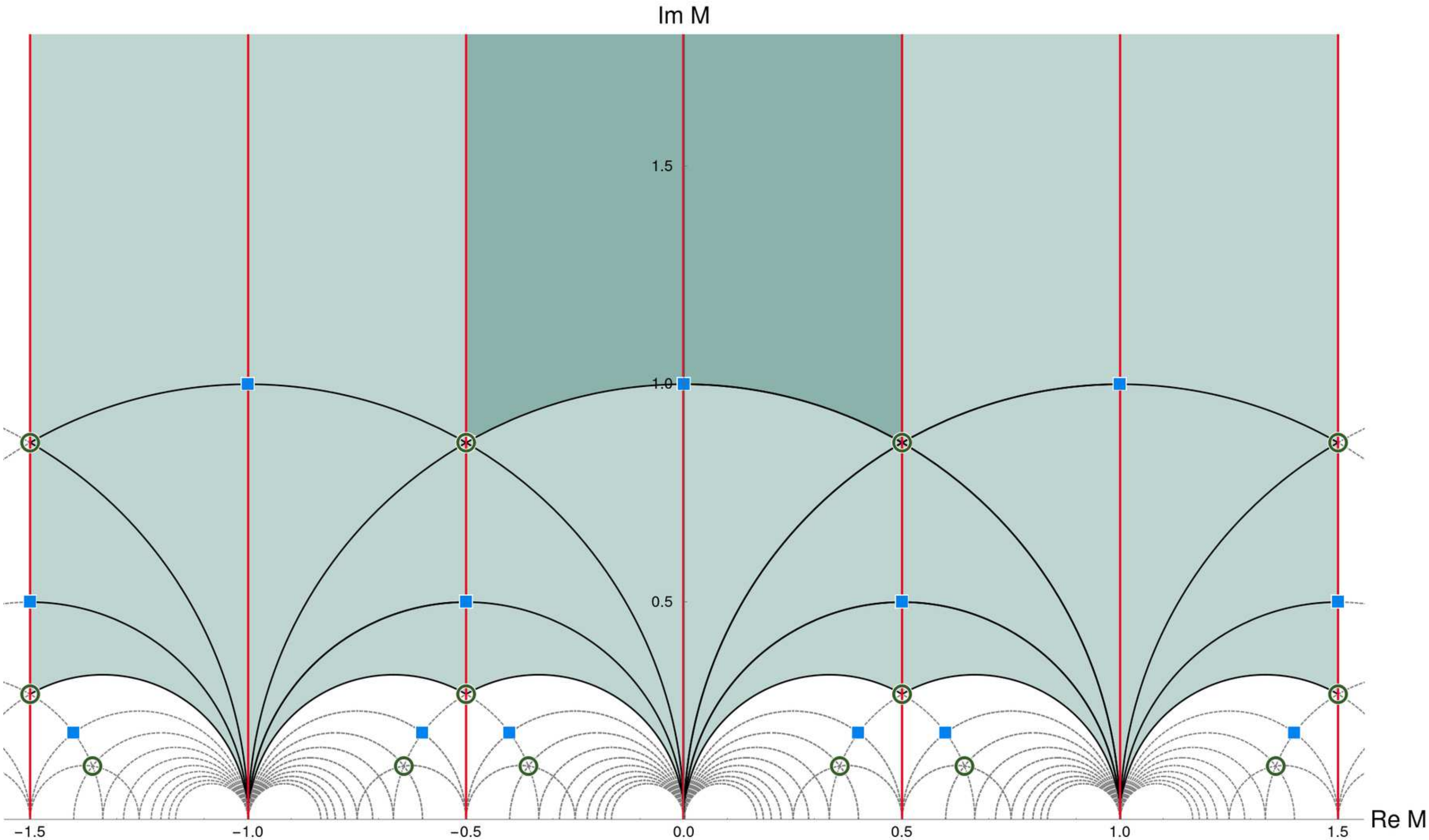
- the traditional flavor group that is universal in moduli space (here $\Delta(54)$)
- the modular flavor group that transforms the moduli nontrivially (here T')

The eclectic flavor group is defined as the multiplicative closure of these groups. Here we obtain for T_2/Z_3

- $\Omega(1) = SG[648, 533]$ from $\Delta(54)$ and $T' = SL(2,3)$
- $SG[1296, 2891]$ from $\Delta(54)$ and $GL(2,3)$ including CP

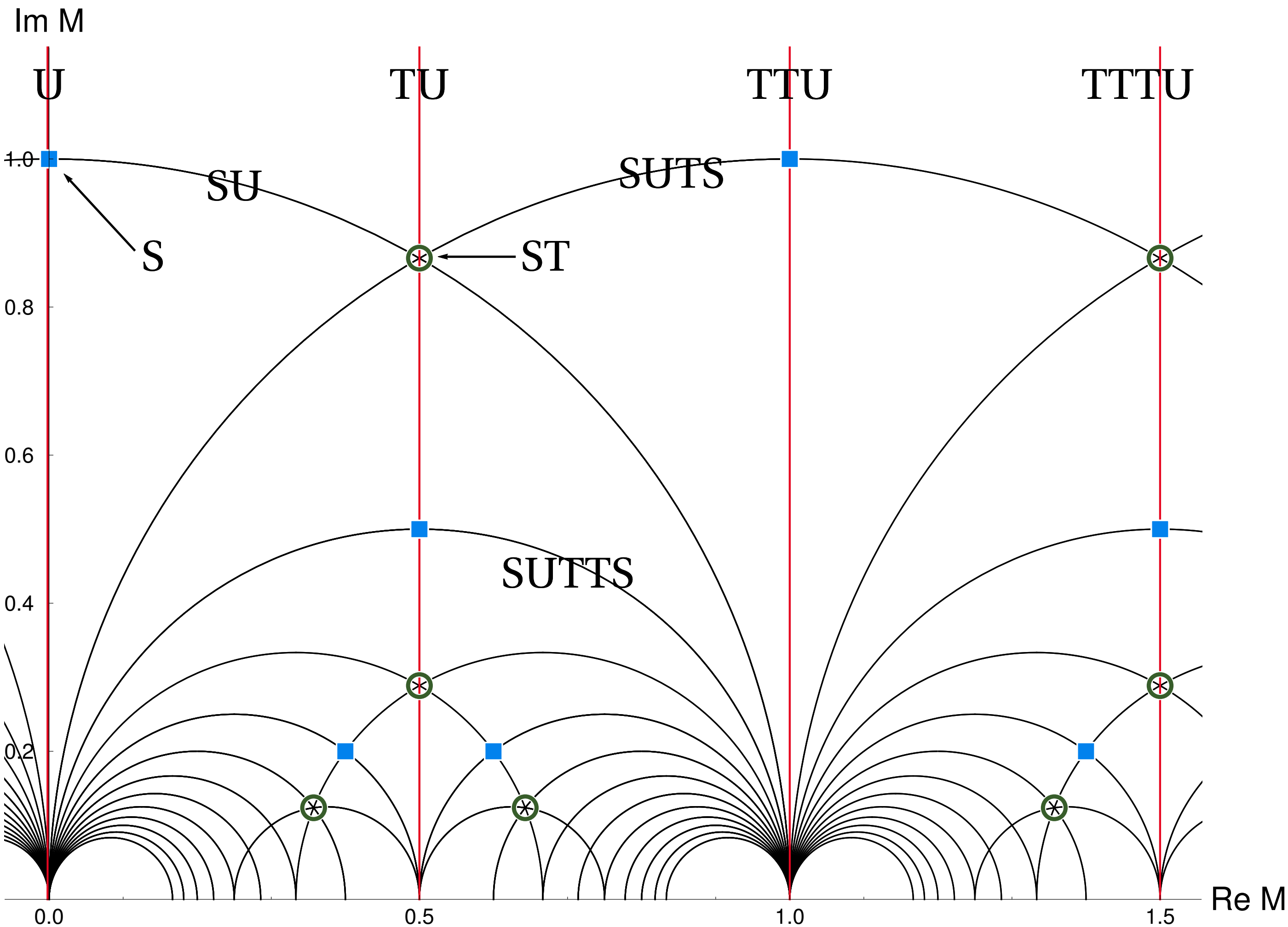
The eclectic group is the largest possible flavor group for the given system, but it is not necessarily linearly realized.

Local Flavor Unification



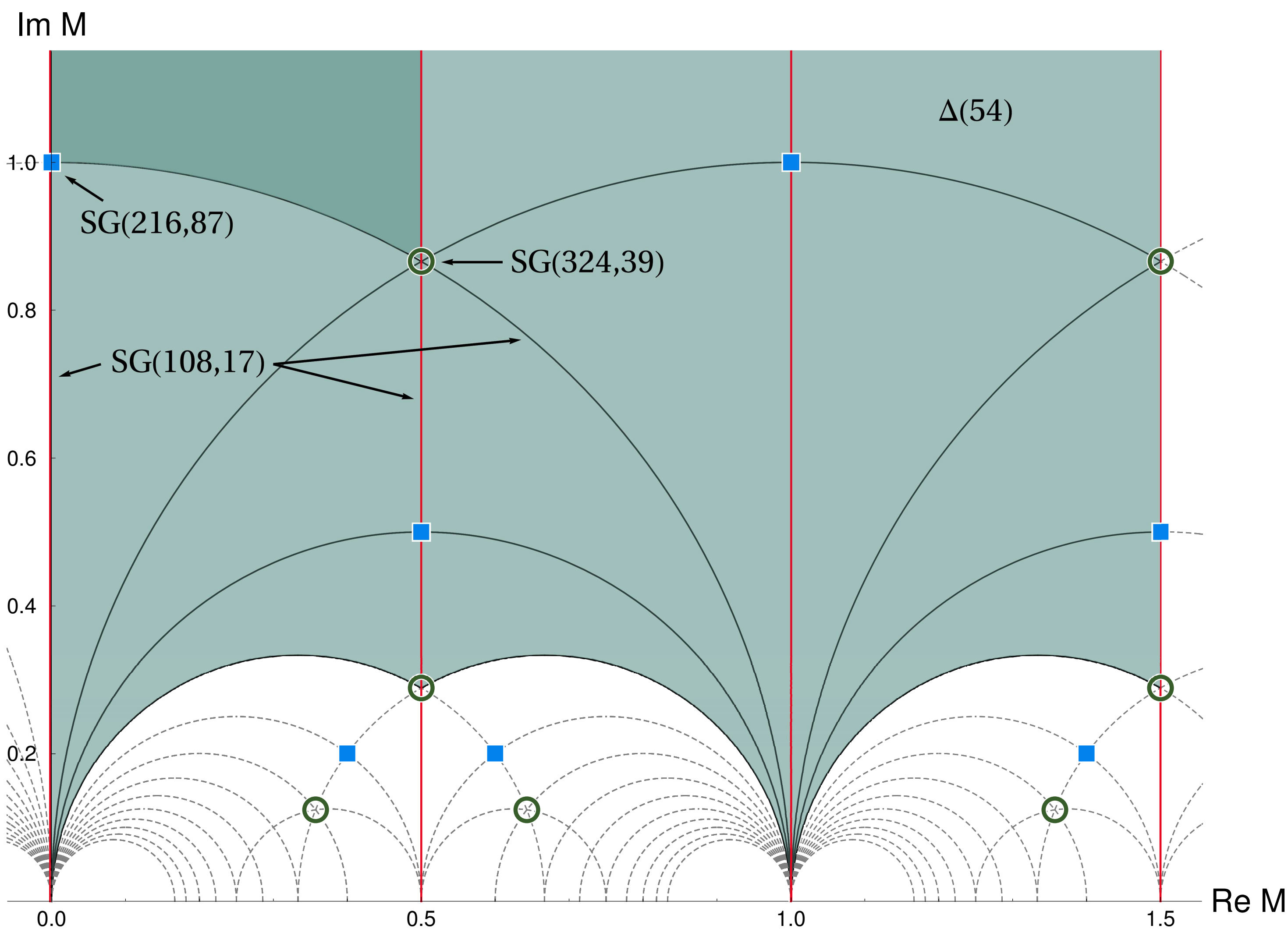
Moduli space of $\Gamma(3)$

Fixed lines and points



$$\mathrm{S} : M \to -\frac{1}{M}, \quad \mathrm{T} : M \to M + 1 \quad \text{and} \quad \mathrm{U} : M \to -\overline{M}$$

Moduli space of flavour groups



Unification of Flavor and CP

Summary of predictions of the string picture:

- traditional flavor symmetries (universal in moduli space)
- modular flavor symmetries and CP are non-universal in moduli space

They unify in the eclectic picture of flavor symmetry.
You cannot just have one without the other.

The non-universality in moduli space leads to

- local flavor unification at specific points in moduli space
- hierarchical structures of masses, mixing angles and phases in vicinity of fixed points or lines
- potentially different pictures for quarks and leptons

Where are we?

So far we have discussed only the simple case of the 2-dimensional Z_3 orbifold with the Kähler modulus (usually called T).
More generally we have to consider also the complex structure modulus U:

- this leads to $SL(2,Z)_T \times SL(2,Z)_U$
- U is frozen in the Z_3 case,
- but still contributes to the eclectic flavor symmetry with R-symmetries, (from compact 6 dimensions) extending $SG[648,533] = \Omega(1)$ to $SG[1944,3448] = \Omega(2)$.
- in the Z_2 case, both T and U are unconstrained
- inclusion of Wilson line leads to Siegel modular group $Sp(2g,Z)$: 3 moduli T, U and Wilson line A for $g = 2$

Top-down model building

First attempts based on realistic orbifold constructions of the heterotic string:

- $Z_3 \times Z_3$-orbifold (severe restrictions in top-down)

  (Carballo-Perez, Peinado, Ramos-Sanchez, 2016)
- flavor groups are $\Delta(54)$ and T'
- breakdown of T' via modulus
- need flavon fields to break $\Delta(54)$ as well
- interplay of breakdown of $\Delta(54)$ and T' (proximity of modulus to fixed points) can create hierarchies
- large angles in lepton sector and see-saw mechanism favour modular flavor symmetry
- quarks with small mixing angles need more structure

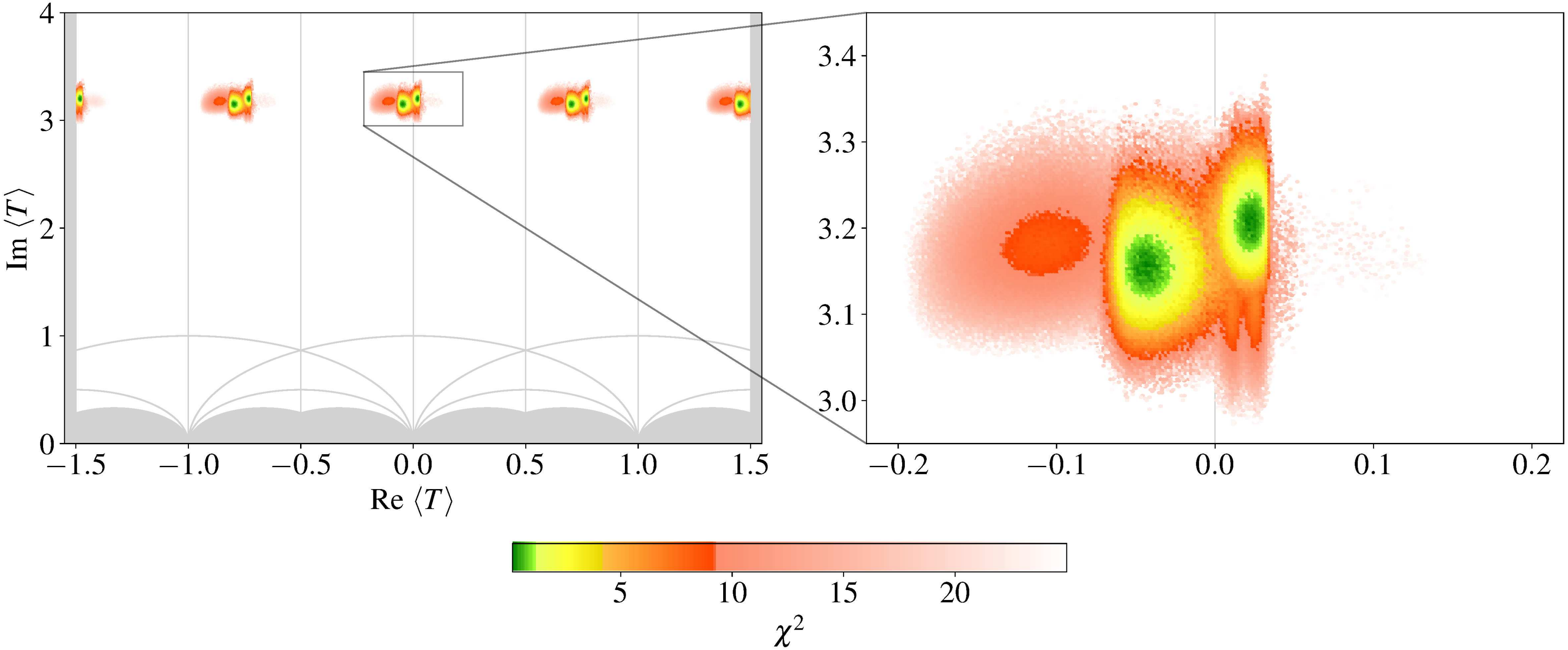
Top-down model building II

First attempts based on simplest class of models of $Z_3 \times Z_3$-orbifold with flavor groups are $\Delta(54)$ and T'

(Baur, Nilles, Ramos-Sanchez, Trautner, 2022)

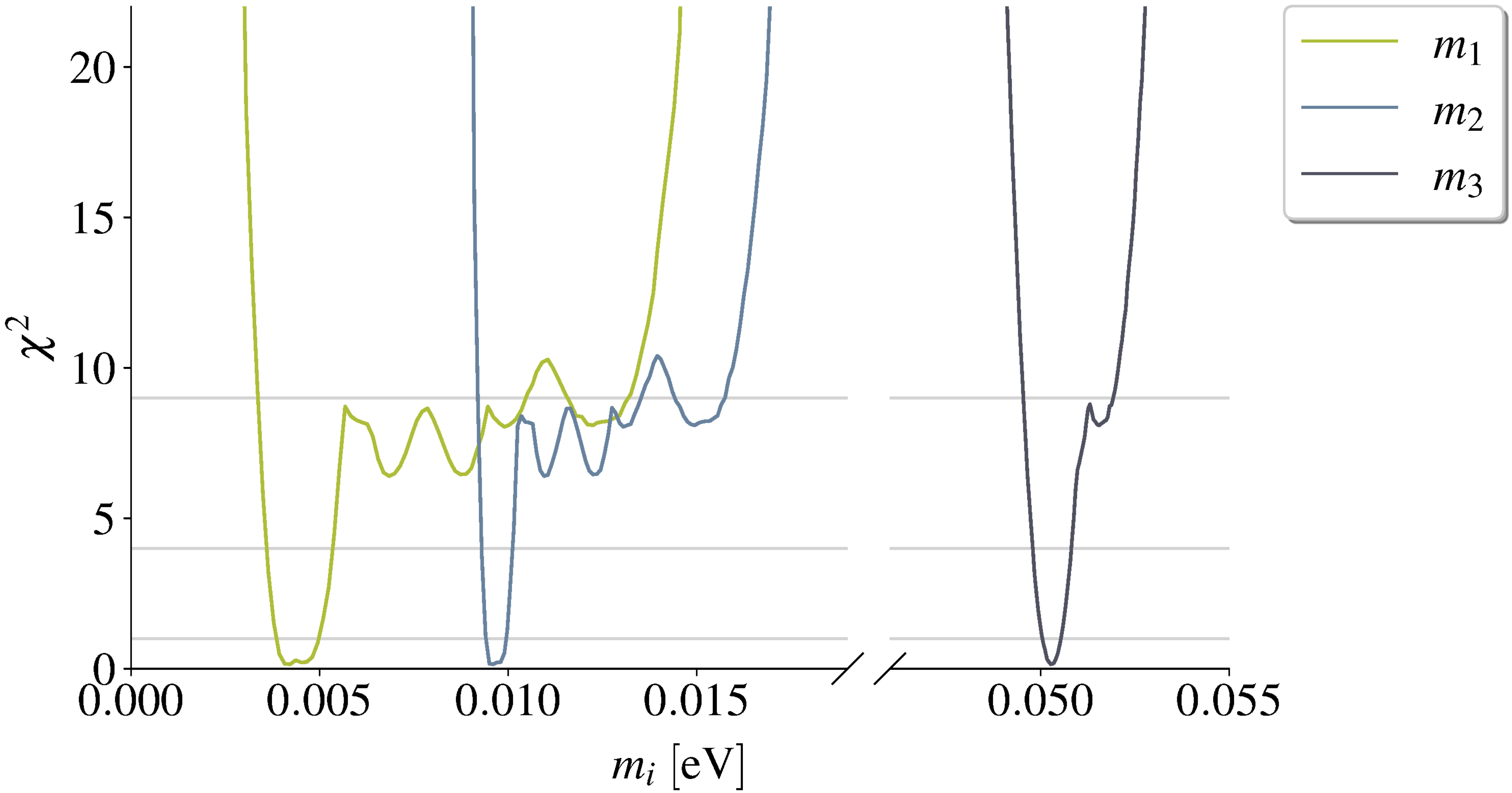
- representations 3_2 of $\Delta(54)$, $1 + 2'$ of T' and $k = -2/3$ (note differences to bottom-up (BU) approach)
- predicts see-saw mechanism in lepton sector
- predicts normal hierarchy for neutrino masses
- severe restrictions on super + Kähler potential
- Kähler potential controlled via traditional flavor sym.
- quark sector needs nontrivial Kähler corrections
- hierarchies appear from a subtle interplay between flavon alignment and breakdown via moduli

Moduli close to fixed points



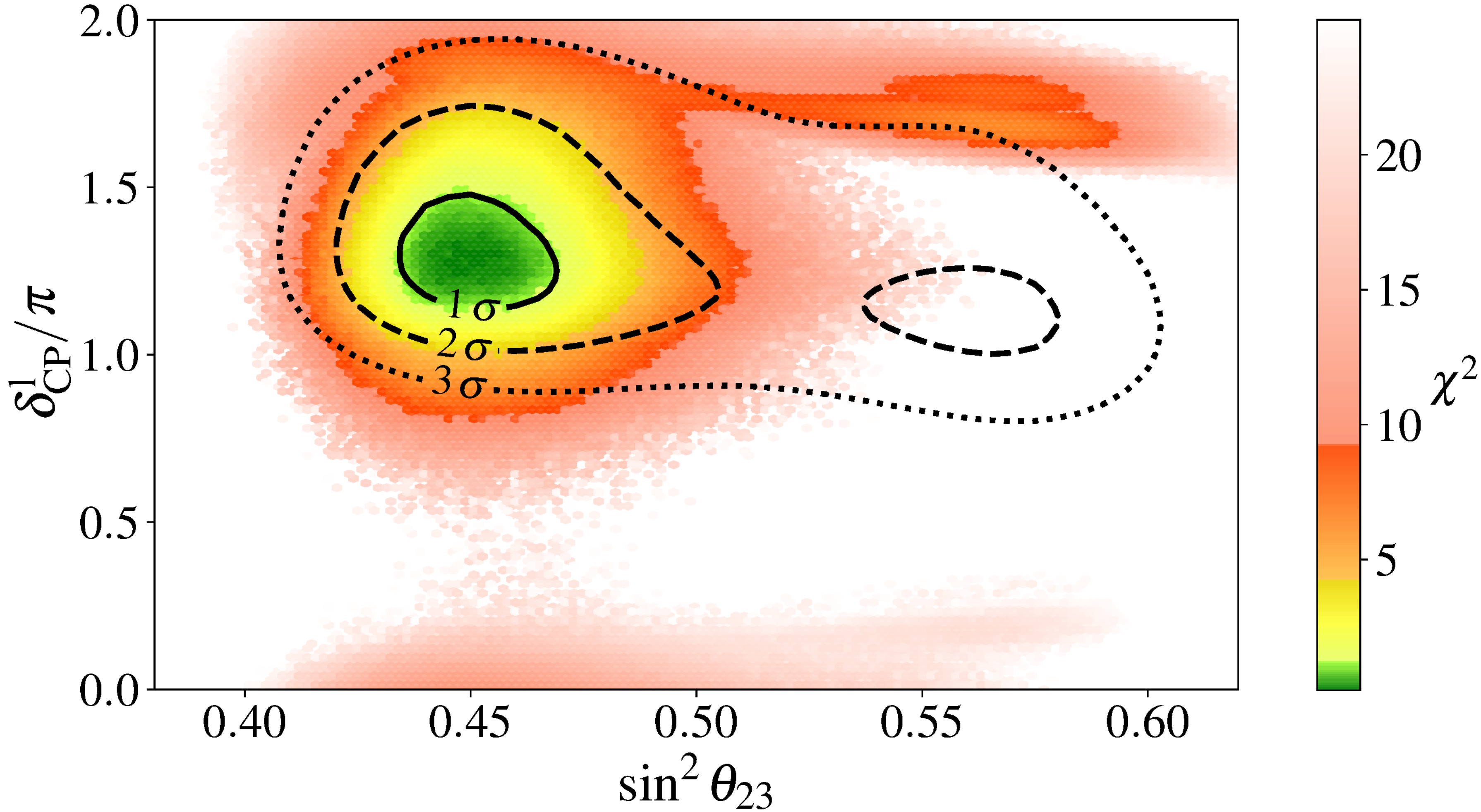
Good fit for Re(T) close to zero and Im(T) close to "∞"

Neutrino Masses

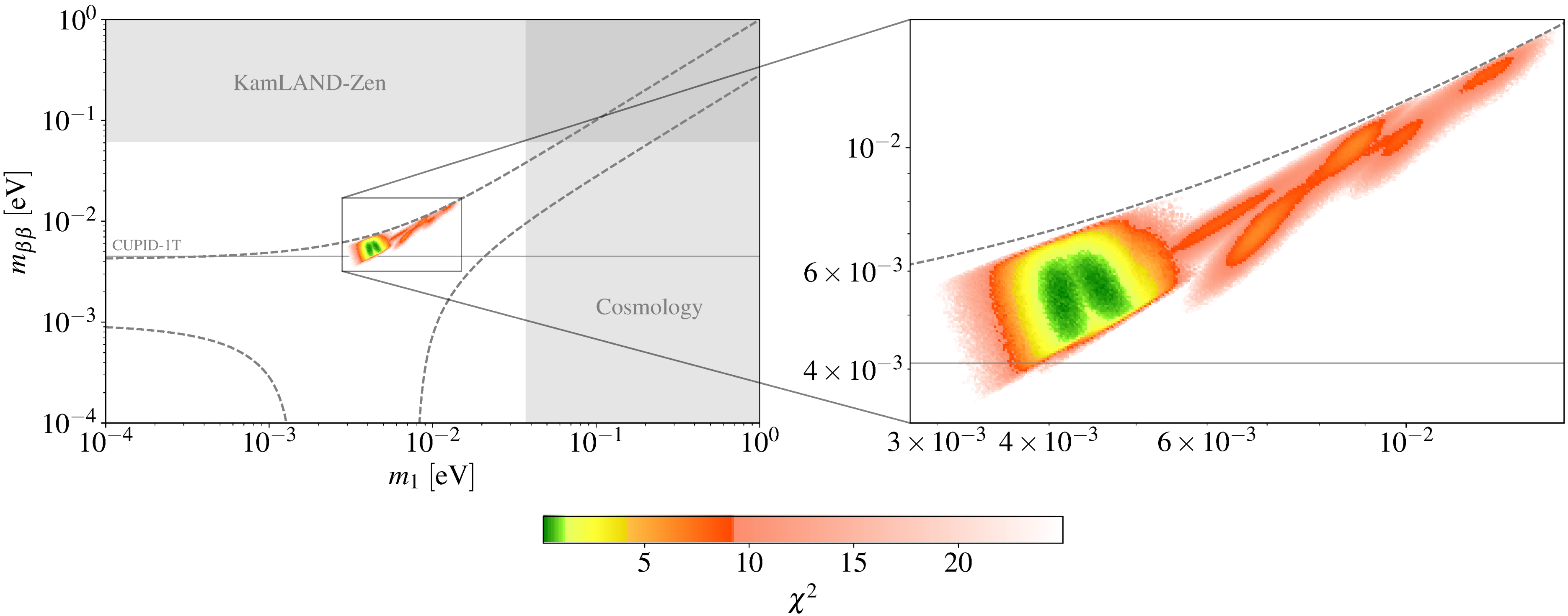


Normal hierarchy for neutrino masses

Mixing Angle and CP



Majorana Masses



High value of effective Majorana mass

Messages

The top-down approach to flavor symmetries leads to a

- unification of traditional (discrete) flavor, CP and modular symmetries within an eclectic flavor scheme
- modular flavor symmetry is a prediction of string theory
- traditional flavor symmetry is universal in moduli space
- there are non-universal enhancements (including CP at some places (broken in generic moduli space))
- CP is a natural consequence of the symmetries of the underlying string theory
- the potential flavor groups are large and non-universal
- spontaneous breakdown as motion in moduli space

Outlook

This opens up a new arena for flavor model building and connections to bottom-up constructions:

- need more explicit string constructions
- role of traditional versus modular symmetries
- the concept of local flavor symmetries allows different flavor groups for different sectors of the theory, as for example quarks and leptons
- but it is not only the groups but also the representations of matter fields that are relevant. Not all of the possible representations appear in the massless sector.

There is still a huge gap between "top-down and bottom-up" constructions

Summary

String theory provides the necessary ingredients for flavor:

- traditional flavor group
- discrete modular flavor group
- a natural candidate for CP
- the concept of local flavour unification

The eclectic flavor group provides the basis:

- this includes a non-universality of flavor symmetry in moduli space
- allows a different flavor structure for quarks and leptons

Back-up: Z_2 orbifold: two moduli

Here the twist does not constrain the moduli T and U

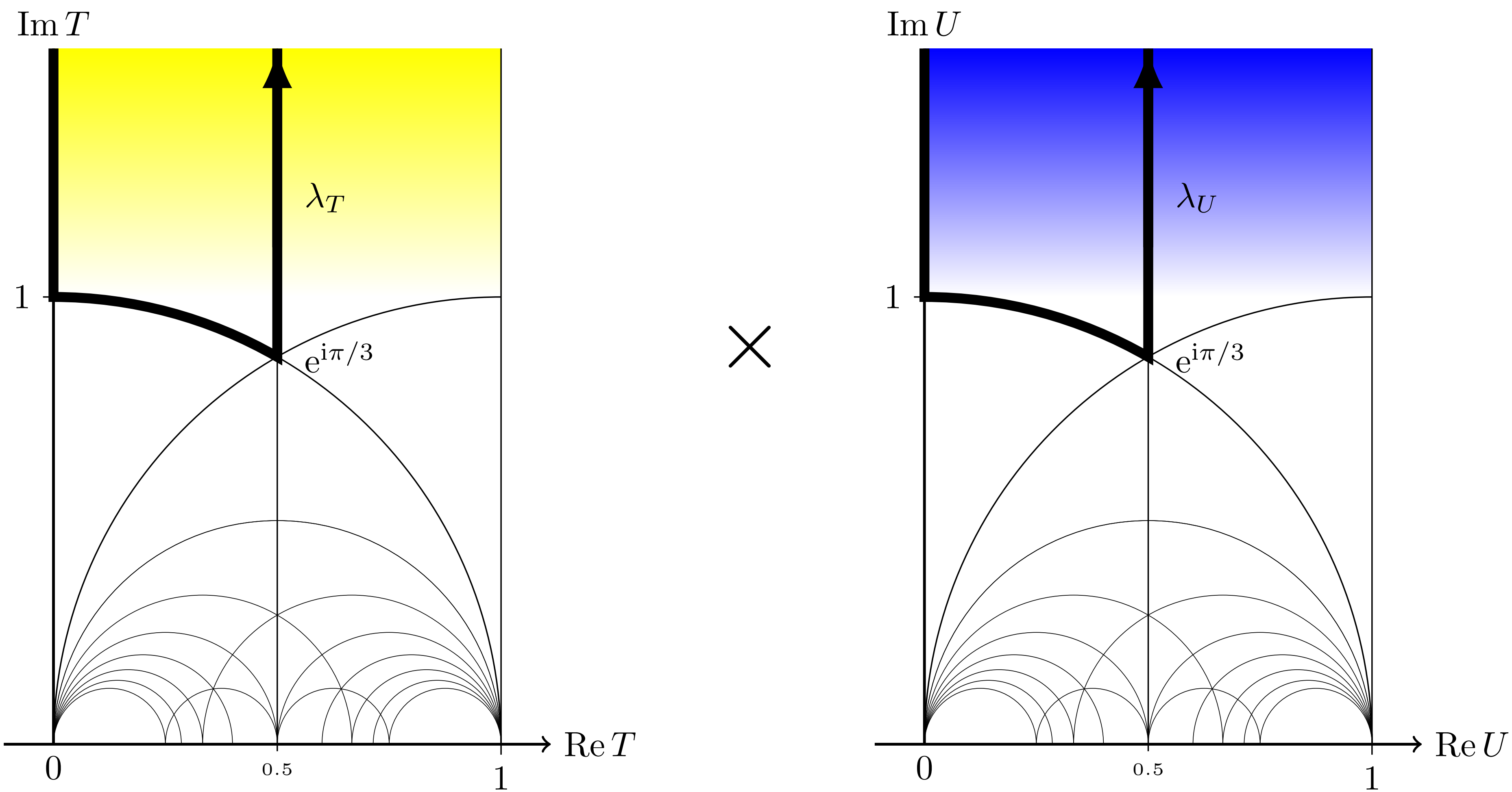
- and we have the full $SL(2,Z)_T \times SL(2,Z)_U$.
- The discrete modular group is $\Gamma_2 \times \Gamma_2 \times Z_2$,
- where $\Gamma_2 = S_3$ and
- Z_2 interchanges T and U (known as mirror symmetry).
- The traditional flavor group is the product of $(D_8 \times D_8)/Z_2$ and a Z_4 R-symmetry.

This leads to an

- eclectic group with 2304 elements (excluding CP)
- or 4608 elements (including CP)

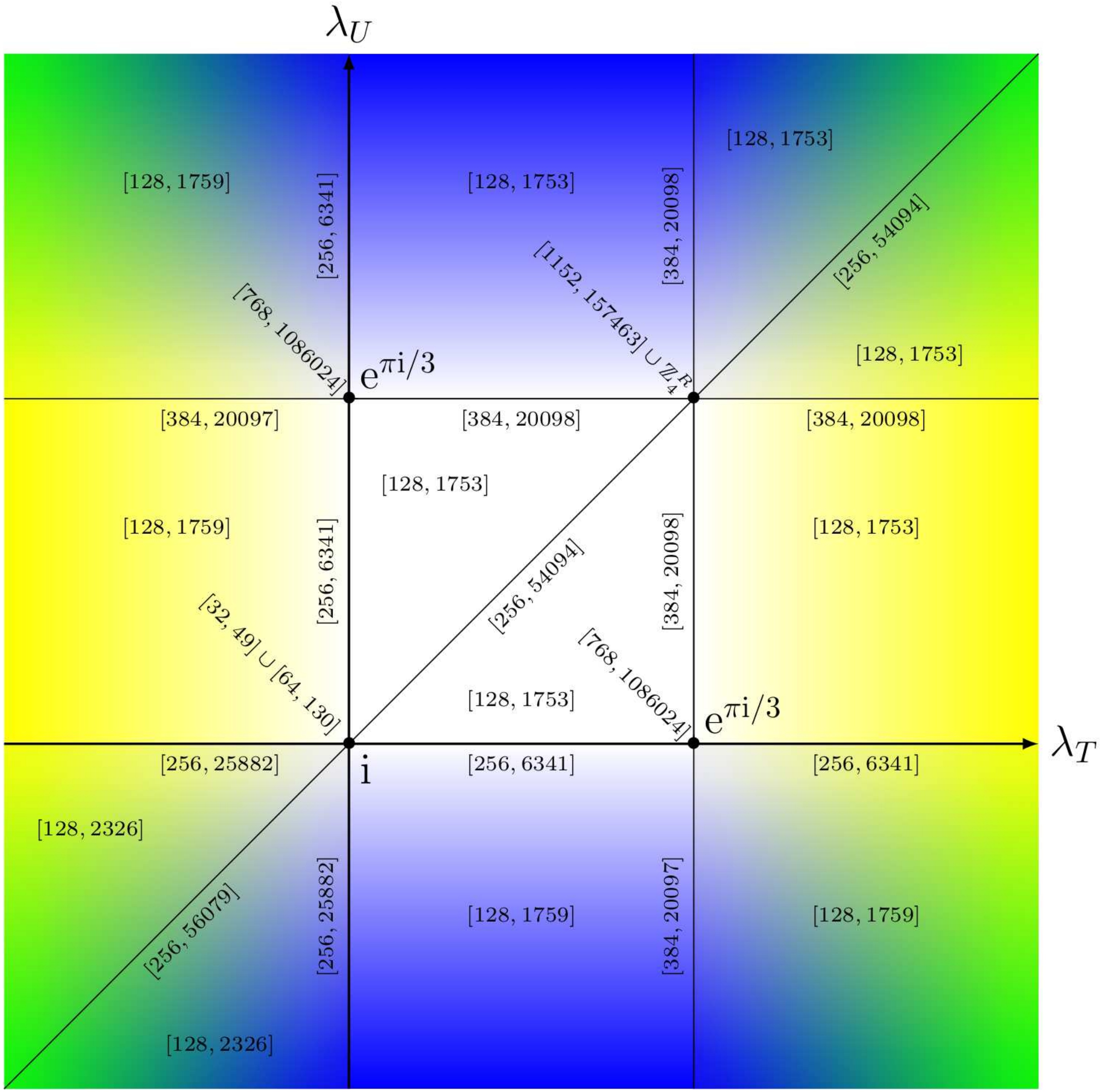
with a rich pattern of local flavor group enhancements.

Z_2-orbifold

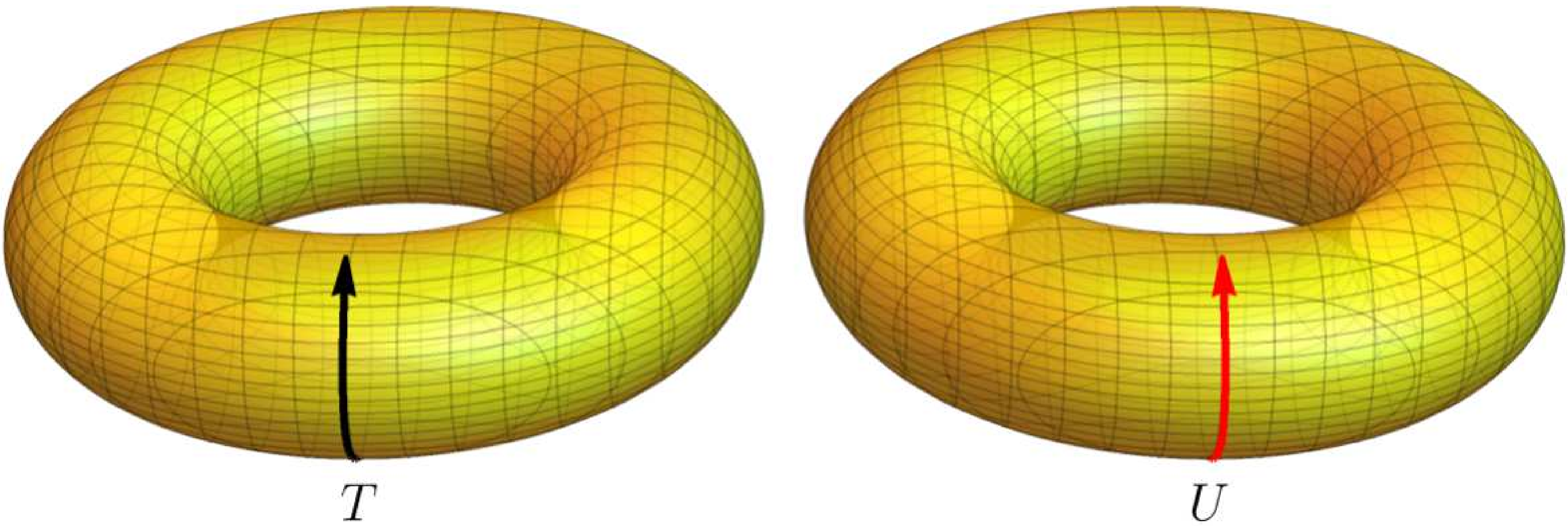


Here we have two unconstrained moduli: T and U

Enhancement

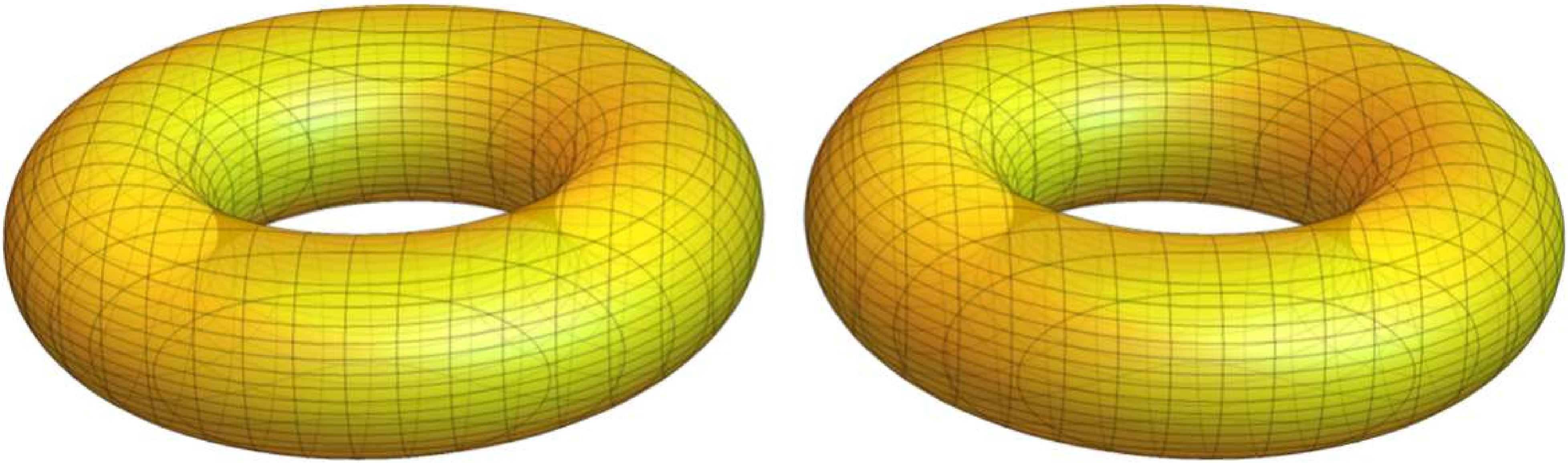


Auxiliary Surface: Double Torus



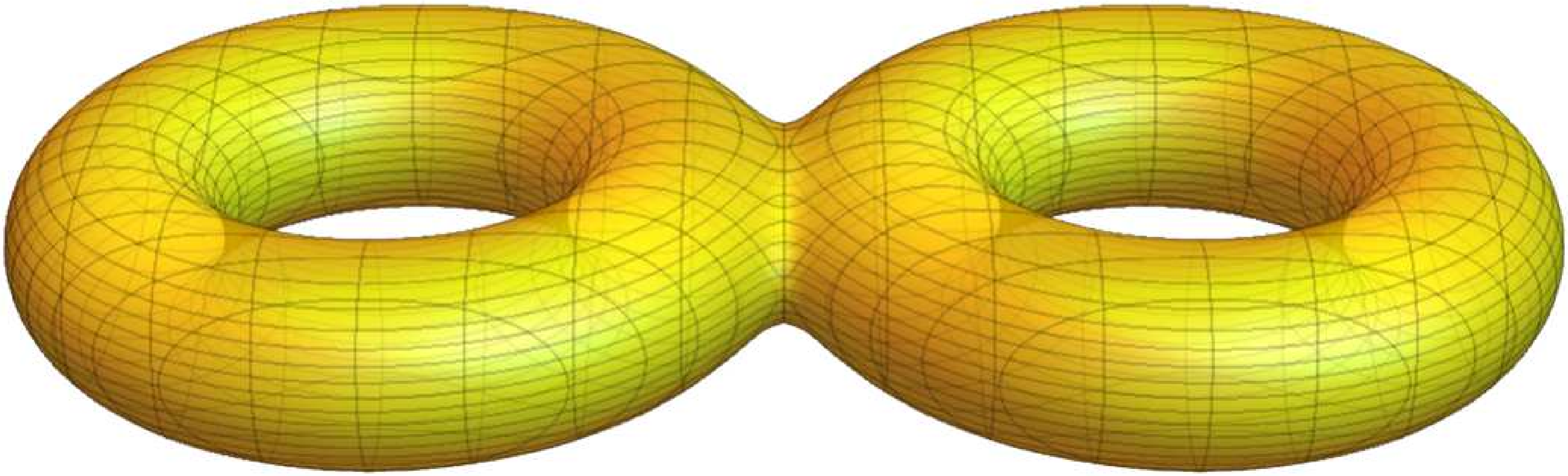
Auxiliary surface for two moduli: $SL(2,Z)_T \times SL(2,Z)_U$

Riemann surface of genus 2



Auxiliary surface for two moduli: T and U

Riemann Surface of Genus 2



Auxiliary surface with three moduli: T + U + Wilson line

Siegel Modular Forms

This leads to a generalization of the modular group to larger groups $Sp(2g, Z)$ characterized through Riemann surfaces of higher genus g:

- for $g = 2$ the Siegel modular group $Sp(4, Z)$
- includes $SL(2, Z)_{U,T}$ and describes three moduli.
- Fundamental domain (6 points, 5 lines, 2 surfaces)
- Orbifold twists are connected to fixed loci in fundamental domain
- Discrete modular group $\Gamma_{g,k}$ ($\Gamma_{1,k} = \Gamma_k$)
- $\Gamma_{2,2} = S_6$ includes $S_3 \times S_3$ and mirror symmetry
- $\Gamma_{2,3}$ has already 51840 elements