Symmetry breaking: Pion as a Nambu-Goldstone boson

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Structure

1. General theory of sponatneous symmetry breaking
   - What it is
   - Spontaneous breaking of a global continuous symmetry (classical)
   - Spontaneous breaking of a global continuous symmetry (QFT)
   - Quantum fluctuations
2. Pion as a Nambu-Goldstone Boson
   - Weak decays
   - Pion as a Nambu-Goldstone boson
3. Summary

What it is

A symmetry is said to be **spontaneously broken**, if the Lagrangian exhibits a symmetry but the ground state/vacuum does not have this symmetry.

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Goldstone-theorem

If the Lagrangian is invariant under a **continuous global** symmetry operation $g \in G$ and the vacuum is invariant under a subgroup $H \subset G$, then there exist $n(G/H) = n(G) - n(H)$ **massless spinless particles**.

With n(G) the number of generators of the group G.

Spontaneous breaking of a global continuous symmetry (classical)

$$\mathcal{L}(\Phi_i, \partial\Phi_i) = \frac{1}{2}(\partial_\mu \Phi_i)^2 - V(\Phi_i)$$

with $V(\Phi_i)$invariant under a **global continous** symmetry group G i.e.

$$V(\Phi_i) = V(\rho(g)\Phi_i) \quad \text{with} \quad g \in G$$
$$\Phi_i \mapsto \Phi_i' = \rho(g)\Phi_i \approx \Phi_i + \delta\Phi_i = \Phi_i + i\epsilon_a T^a \Phi_i$$

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We demand that the vacuum has a **minimal energy**
$\Rightarrow$ minimize V

$$\Phi_{i,\mathrm{min}} =< 0|\Phi_i|0 >= const. \qquad \chi_i := \Phi_i - \Phi_{i,\mathrm{min}}$$
$$V(\Phi_i) = V(\Phi_{i,\mathrm{min}}) + \frac{\partial V}{\partial \Phi_i}(\Phi_{i,\mathrm{min}})\chi_i + \frac{1}{2}\frac{\partial^2 V}{\partial \Phi_i \partial \Phi_j}\chi_i\chi_j + \ldots$$

Spontaneous breaking of a global continuous symmetry (classical)

$V(\Phi_{i,\min})$ is a minimum, therefore

$$\frac{\partial V}{\partial \Phi_i}(\Phi_{i,\min}) = 0$$

$$\frac{\partial^2 V}{\partial \Phi_i \partial \Phi_j} := M_{ij}^2 \quad \text{is semi-positive definite}$$

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Now use the **symmetry**:

$$V(\Phi_{i,\mathrm{min}}) = V(\rho(g)\Phi_{i,\mathrm{min}}) = V(\Phi_{i,\mathrm{min}}) + \frac{1}{2} M^2_{ij} (\delta\Phi_{i,\mathrm{min}})(\delta\Phi_{j,\mathrm{min}})$$
$$\Rightarrow 0 = M^2_{ij} \delta\Phi_{j,\mathrm{min}} \quad \forall i$$
$$\Rightarrow 0 = M^2_{ij} T^a \Phi_{j,\mathrm{min}} \quad \forall i$$

Spontaneous breaking of a global continuous symmetry (classical)

important result

$$0 = M_{ij}^2 T^a \Phi_{j,\min} \qquad \forall i$$

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$$T^a \in H \subset G \qquad \text{with}$$
$$T^a \Phi_{j,\mathrm{min}} = 0$$

Every generator of H leaves the vacuum invariant.

Spontaneous breaking of a global continuous symmetry (classical)

important result

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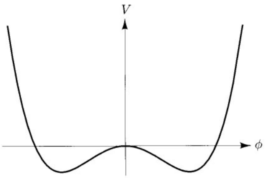
1. $$T^a \in H \subset G \quad \text{with}$$
   $$T^a \Phi_{j,\mathrm{min}} = 0$$
   Every generator of H leaves the vacuum invariant.

2. $$T^a \notin H \Rightarrow T^a \Phi_{j,\mathrm{min}} \neq 0$$
   $\Rightarrow T^a \Phi_{j,\mathrm{min}}$ is an **eigenstate of** M^2 with **0** eigenvalue
   $\Rightarrow T^a \Phi_{j,\mathrm{min}}$ is a **Nambu-Goldstone boson**

The number of Nambu-Goldstone bosons is n(G)-n(H)=n(G/H).

Pictures



$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4$

with $\phi_{\rm min} = \sqrt{\frac{6}{\lambda}}\mu^2$

$\phi = \phi_{\rm min} + \sigma$

$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(2\mu^2)\sigma^2 - \sqrt{\frac{\lambda}{6}}\mu\sigma^3 - \frac{\lambda}{4!}\sigma^4$

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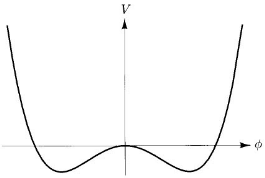

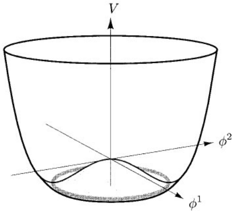

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Massive modes correspond to fluctuations in **radial** direction.

Massless modes correspond to fluctuations in **angular** direction.

Spontaneous breaking of a global continuous symmetry (QFT)

In QFT every **continuous symmetry** implies the existence of a conserved current $J^\mu(x)$:

$$\partial_\mu J^\mu(x) = 0 \quad \longrightarrow \quad Q = \int d^D x J^0(\vec{x}, t) \quad \text{is conserved, i.e.}$$

$$[H, Q] = 0$$

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$$[H, Q] = 0$$

- $Q|0> = 0$, for every energy-eigenstate $|\Phi >= \Phi|0>$ define

$$|\Phi^{'} > := e^{i\Theta Q}\Phi e^{-i\Theta Q}|0>$$

$$H|\Phi^{'} > = He^{i\Theta Q}\Phi e^{-i\Theta Q}|0 >= He^{i\Theta Q}\Phi|0>$$

$$= e^{i\Theta Q}H\Phi|0 >= e^{i\Theta Q}E\Phi|0 >= E|\Phi^{'} >$$

Energy spectrum is organized in **multiplets** of degenerate states.

Spontaneous breaking of a global continuous symmetry (QFT)

- $Q|0> \neq 0$ Problem, Q is not defined

$$
\begin{aligned}
<0|Q^2(t)|0> &= \int d^D x <0|J^0(\vec{x},t)Q(t)|0> \\
&= \int d^D x <0|e^{i\vec{P}\cdot\vec{x}} J^0(\vec{0},0) e^{-i\vec{P}\cdot\vec{x}} Q(t)|0> \\
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$$

But the **commutator with Q** may be well defined.
Current conservation implies:

$$
0 = \int d^D x [\partial^\mu J_\mu(\vec{x},t), \Phi(0)] = \partial^0 \int d^D [J^0(\vec{x},t), \Phi(0)] + \int d\vec{S} \cdot [\vec{J}(\vec{x},t), \Phi(0)]
$$

Spontaneous breaking of a global continuous symmetry (QFT)

$$\Rightarrow \frac{d}{dt}[Q(t), \Phi(0)] = 0$$

The **symmetry is broken**, if there is an operator Φ s.t.:

$$< 0|[Q(t), \Phi(0)]|0 >= \eta \neq 0$$

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then:

$$\eta = \sum_n \int d^D x \{< 0|J_0(x)|n > < n|\Phi(0)|0 > - < 0|\Phi(0)|n >< n|J_0(x)|0 >\}$$

$$= \sum_n (2\pi)^D \delta^D(\vec{p_n})\{< 0|J_0(0)|n > < n|\Phi(0)|0 > e^{-iE_n t} - < 0|\Phi(0)|n >< n|J_0(0)|0 > e^{iE_n t}\}$$

Since this is time independent, there has to be a state with:

Nambu-Goldstone Boson

$$E_n = 0 \quad \text{for} \quad \vec{p}_n = 0 \quad \Rightarrow m_n = 0$$
$$< n|\Phi(0)|0 > \neq 0$$
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So the Nambu-Goldstone boson has to carry all the **quantum numbers of the conserved current**.

In general there will be **n(G) currents** J^a_μ. Of these **n(H)** generate the **symmetry group of vacuum**. So again there are n(G/H)=n(G)-n(H) Nambu-Goldstone bosons.

Quantum fluctuations

Can the massless fields wander away from their ground state?
Calculate mean square fluctuation:

$$
\begin{aligned}
< b(0)^2 > &= \frac{1}{Z}\int Dbe^{(iS[b])} b(0)b(0) \\
&= \lim_{x\to 0}\frac{1}{Z}\int Dse^{iS(b)} b(x)b(0) \\
&= \lim_{x\to 0}\int \frac{d^d k}{(2\pi)^d}\frac{e^{ik\cdot x}}{k^2}
\end{aligned}
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The UV devergencies can be regulated by a **cutoff**.

But for $d \leq 2$ there is an **IR divergence**.

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Coleman-Mermin-Wagner theorem

Spontaneous symmetry breaking is impossible in $d \leq 2$.

How do pions enter the game?

We want to calculate dacay rates of semileptonic decays like:

$$n \longrightarrow p + e^- + \bar{\nu}$$
$$\pi^- \longrightarrow \pi^0 + e^- + \bar{\nu}$$

Use an effective Lagrangian

$$\mathcal{L} = G[\bar{e}\gamma^\mu(1-\gamma_5)\nu](J_\mu - J_{\mu 5})$$

Where J_μ and $J_{\mu 5}$ are hadronic currents which include **strong interaction effects**.
We have to calculate $< p|J_\mu(x) - J_{\mu 5}(x)|n >$ or $< 0|J_\mu(0) - J_{\mu 5}(0)|\pi^- >$

Nucleon β-decay

$$
\begin{aligned}
< p(k^{'})|J_{\mu 5}(x)|n(k) > &=< k^{'}|e^{iP\cdot x}J_{\mu 5}(0)e^{-iP\cdot x}|k > \\
&=< k^{'}|J_{\mu 5}(0)|k > e^{i(k^{'}-k)\cdot x}
\end{aligned}
$$

Use **Lorentz invariance** to simplify $< k^{'}|J_{\mu 5}(0)|k >$ with $q := k^{'} - k$

$$
\begin{aligned}
< k^{'}|J_{\mu 5}(0)|k > = \bar{u}_p(k^{'})[&-i\gamma_\mu\gamma_5\mathbf{F}^{'}(q^2) + q_\mu\gamma_5\mathbf{G}(q^2) \\
&+ (k^{'}_\mu + k_\mu)\gamma_5\mathbf{F}^{'}_2(q^2) + i[\gamma_\mu,\gamma_\nu]\gamma_5 q^\nu\mathbf{F}^{'}_3(q^2)]u_n(k)
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\end{aligned}
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And use the **Gordan-Identity**:

$$
\begin{aligned}
\bar{u}_p(k^{'})[i\frac{1}{2}[\gamma_\mu,\gamma_\nu]\gamma_5 q^\nu]u_n(k) = \bar{u}_p(k^{'})[i(k^{'}_\mu + k_\mu)\gamma_5 \\
+ (m_p - m_n)\gamma_\mu\gamma_5]\bar{u}_n(k)
\end{aligned}
$$

Nucleon β-decay

So if we define:

$$\mathbf{F}_3(q^2) = \mathbf{F}'_3(q^2) + \frac{1}{2i}\mathbf{F}'_2(q^2)$$

$$\mathbf{F}(q^2) = \mathbf{F}'(q^2) - (m_p - m_n)\mathbf{F}'_2(q^2)$$

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we obtain:

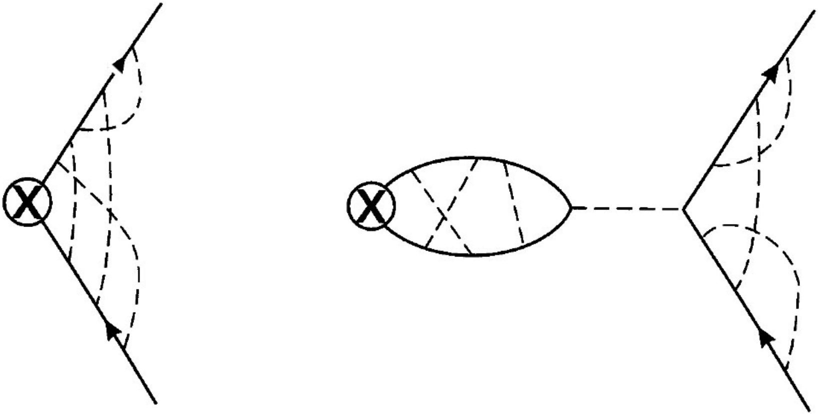
Decomposition of $< k^{'}|J_{\mu 5}(0)|k >$ into Form factors

$$< k^{'}|J_{\mu 5}(0)|k >= u_p(k^{'})[-i\gamma_\mu\gamma_5\mathbf{F}(q^2) + q_\mu\gamma_5\mathbf{G}(q^2) + i[\gamma_\mu,\gamma_\nu]\gamma_5 q^\nu \mathbf{F}_3(q^2)]u_n(k)$$

And similarly we obtain:

$$< 0|J_5^\mu(0)|\pi(k) >= iF_\pi k^\mu$$

Problems with strongly interacting particles



In general a pertubation theory is **not well defined**.

Breaking of chiral symmetry

$$M_\pi \approx 139MeV << 938Mev \approx m_N \Rightarrow M_\pi \approx 0$$

Is it possible that the **pion** is a **Nambu-Goldstone Boson**?

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Suppose $\partial_\mu J_5^\mu(x){=}0$ i.e. **chiral symmetry** holds then:

$$< 0|J_5^\mu(0)|\pi(k) >= iF_\pi k^\mu \quad \Leftrightarrow \quad < 0|J_0(0)|n > \neq 0$$

$$< \pi|\pi(0)|0 >= 1 \quad \Leftrightarrow \quad < n|\Phi(0)|0 > \neq 0$$

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Pion as a Nambu-Goldstone Boson

The Pion is a Nambu-Goldstone Boson in a world in which chiral symmetry holds.

Is the Pion massless?

If chiral symmetry holds then:

$$-ik_\mu < 0|J_5^\mu(0)|\pi(k) > \exp(-ik\cdot x) = \partial_\mu < 0|J_5^\mu(0)|\pi(k) > \exp(-ik\cdot x)$$
$$= < 0|\partial_\mu J_5^\mu(x)|\pi(k) > = 0$$

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$$= < 0|\partial_\mu J_5^\mu(x)|\pi(k) > = 0$$

$$k^\mu < 0|J_5^\mu(0)|\pi(k) > = iF_\pi k^\mu k_\mu = iF_\pi M_\pi^2$$

Thus $\partial_\mu J_5^\mu(x) = 0$ implies that $M_\pi^2 = 0$

Goldberger-Treiman relation

$$q^\mu < k^{'}|J_{\mu 5}(0)|k> = -i < k^{'}|\partial^\mu J_{\mu 5}(x)|k> \exp(-iq\cdot x) = 0$$

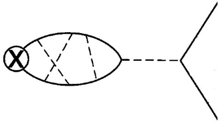
$$\begin{aligned} q^\mu < k^{'}|J_{\mu 5}(0)|k> &= q^\mu \bar{u}_p(k^{'})[-i\gamma_\mu\gamma_5 \mathbf{F}(q^2) + q_\mu \gamma_5 \mathbf{G}(q^2) \\ &+ i[\gamma_\mu,\gamma_\nu]\gamma_5 q^\nu \mathbf{F}_3(q^2)]u_n(k) \\ &= \bar{u}_N(k^{'})[(k^{'\mu}(-i)\gamma_\mu\gamma_5\mathbf{F}(q^2) - k^\mu(-i)\gamma_\mu\gamma_5\mathbf{F}(q^2) \\ &+ q^2\gamma_5\mathbf{G}(q^2) + i[\gamma_\mu,\gamma_\nu]\gamma_5 q^\nu q^\mu \mathbf{F}_3(q^2)]u_N(k) \\ &= \gamma_5[2m_N\mathbf{F}(q^2) + q^2\mathbf{G}(q^2)] = 0 \end{aligned}$$

by using the **Dirac equation**:

$$\bar{u}_N(k^{'})(k^{'}_\mu\gamma^\mu - im_N) = (k_\mu\gamma^\mu - im_N)u_N(k) = 0$$

But if $q \to 0$ this implies $m_N = 0$!?!

Goldberger-Treiman relation



this diagram gives a contribution
$-iF_\pi q^\mu \frac{i}{q^2} G_{\pi NN} \bar{u}_N(k') \gamma_5 u_N(k)$

Note that there are still **infinitely many diagrams** which have a pole at $q^2 = 0$

$$G(q^2) \sim F_\pi \frac{1}{q^2} G_{\pi NN} \quad \text{for} \quad q \to 0$$

Goldberger-Treiman relation

$$G_{\pi NN} = \frac{2m_N g_A}{F_\pi} \quad \text{with} \quad g_A = -F(0)$$

Experimental test of Goldberger-Treiman relation

$$
\begin{aligned}
m_N &= \frac{(m_p + m_n)}{2} = 939.9\text{MeV} \\
g_A &= 1,257 \\
F_\pi &= 93\text{MeV} \\
G_{\pi NN}^{GTR} &\approx 25.4 \\
G_{\pi NN} &= 27.0
\end{aligned}
$$

In good agreement with the experiment.

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Please note that this result is quite **general**.
No assumptions about the broken symmetry have been made.

More about chiral symmetry

The Lagrangian of strong interaction:

$$\mathcal{L} = -\bar{u}\gamma^{\mu} D_{\mu} u - \bar{d}\gamma^{\mu} D_{\mu} d + m_u(\bar{u}_R u_L + \bar{u}_L u_R) + m_d(\bar{d}_L d_R + \bar{d}_R d_L) + \ldots$$

has a $SU(2)_V \otimes SU(2)_A$ symmetry in the case $m_u = m_d = 0$.

$$q \mapsto q' = \exp(i\vec{\Theta}_V \cdot \vec{\tau} + i\gamma_5 \vec{\Theta}_A \cdot \vec{\tau})q$$

with $\vec{\tau}$ the isospin (Pauli) matrices.

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with $\vec{\tau}$ the isospin (Pauli) matrices.
And the **conserved currents**:

$$\vec{J}^\mu = i\bar{q}\gamma^\mu\vec{\tau}q \quad \text{and} \quad \vec{J}_5^\mu = i\bar{q}\gamma_\mu\gamma_5\vec{\tau}q$$

with charges:

$$\vec{Q}_V = \int d^3x\vec{J}^0(\vec{x},t)$$

$$\vec{Q}_A = \int d^3x\vec{J}_5^0(\vec{x},t)$$

More about chiral symmetry

They act in the following way on q:

$$[\vec{Q}_V, q] = -\vec{\tau} q$$
$$[\vec{Q}_A, q] = -\gamma_5 \vec{\tau} q$$

So if the symmetry is unbroken $\vec{Q}_A$ transforms a state |h> into a state of $\vec{Q}_A|h>$ of **opposite parity**.

No such **parity doubling** is observed in the hadron spectrum. Conclude that $SU(2)_V \otimes SU(2)_A$ is broken to $SU(2)_V$ **isospin group**.
The three Pions are the three Nambu-Goldstone Bosons of the three currents $\vec{J}_5^{\mu}$.

Summary/Remarks

- Breaking of a continuous global symmetry leads to **Nambu-Goldstone Bosons**.
- No information about **how** the breaking occurs is needed.
- Natural explanation for **smallness** of **Pion mass**.
- Found a technique to **relate** infinitly many diagramms.

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- Natural explanation for **smallness** of **Pion mass**.
- Found a technique to **relate** infinitly many diagramms.
- From the assumption that Pions and even Kaons and the Eta are Goldstone bosons one can construct **effective field theories** like non-linear sigma model or chiral pertubation theory.
- The QCD Lagrangian for massless Quarks has two other **global symmetries**. One, $U(1)_V$ implies **Baryon-number conservation**. The other one, $U(1)_A$ will be treated **next week**.

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Thank you

Sources

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