Physikalisches Institut
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Theoretische Physik

Exercise 10
08.01.03
WS 02/03

Exercises on 'Elementary Particle Physics'

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1. *Muon Decay (Part I)*

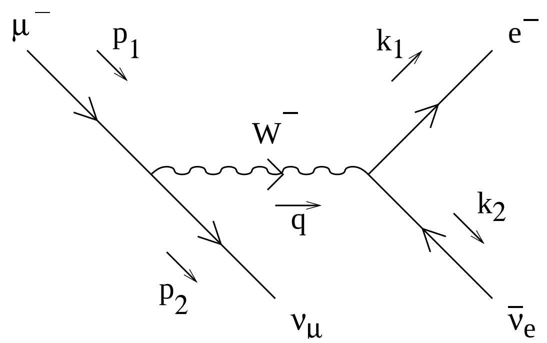
   (a) For later use perform the following calculations:

      i. Show that $\gamma^0(\gamma^\mu(1-\gamma_5))^\dagger\gamma^0 = (1+\gamma_5)\gamma^\mu$.

      ii. Show that $\mathcal{P}_\pm \equiv \frac{1}{2}(\mathbb{1} \pm \gamma_5)$ are projection operators, i.e. $\mathcal{P}_+ + \mathcal{P}_- = \mathbb{1}$, $\mathcal{P}_\pm^2 = \mathcal{P}_\pm$ and $\mathcal{P}_+\mathcal{P}_- = 0$.

   Remember that $\mathrm{tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\rho) = -4i\epsilon^{\mu\nu\sigma\rho}$ for $\epsilon^{0123} = +1$ and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

   Consider the weak process $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$. It's *Feynman* diagram is given by:



   (b) The electroweak *Feynman* rules are very similar to the QED ones. The differences (important here) are:

      i. Concerning the spinors $u(p,s), \bar{u}(p,s), v(p,s)$ and $\bar{v}(p,s)$ treat the μ^- and the ν's 'electron-like', the $\bar{\nu}$ correspondingly 'positron-like'.

      ii. For each vertex now insert a factor $-\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$, where g is the electroweak coupling constant.

      iii. Instead of the photon propagator use the W propagator $\frac{-i(\eta_{\mu\nu}-q_\mu q_\nu/M_W^2)}{q^2-M_W^2}$, where q is the momentum of W and M_W its mass.

   We will work in a limit that the squared momentum transfer q^2 is much smaller than the squared mass M_W^2. Use the electroweak *Feynman* rules to write down the amplitude $\mathcal{M}$ for the process sketched.

(c) We now want to compute $\langle|\mathcal{M}|^2\rangle$ ($\langle \quad \rangle$ just means averaged). Proceed as follows:

i. Write down $|\mathcal{M}|^2$ in the form $L^{\mu\nu}L_{\mu\nu}$.

ii. 'Average' the spins (how?) and use the *Casimir* trick[1] (c.f. Ex 7) in one or more of the following forms:

$$\begin{aligned}
\sum_{\text{all spins}} [\bar{u}(a)\Gamma_1 u(b)][\bar{u}(a)\Gamma_2 u(b)]^* &= \mathrm{tr}[\Gamma_1(\not{p}_b + m_b)\gamma^0\Gamma_2^\dagger\gamma^0(\not{p}_a + m_a)] \; , \\
\sum_{\text{all spins}} [\bar{u}(a)\Gamma_1 v(b)][\bar{u}(a)\Gamma_2 v(b)]^* &= \mathrm{tr}[\Gamma_1(\not{p}_b - m_b)\gamma^0\Gamma_2^\dagger\gamma^0(\not{p}_a + m_a)] \; , \\
\sum_{\text{all spins}} [\bar{v}(a)\Gamma_1 v(b)][\bar{v}(a)\Gamma_2 v(b)]^* &= \mathrm{tr}[\Gamma_1(\not{p}_b - m_b)\gamma^0\Gamma_2^\dagger\gamma^0(\not{p}_a - m_a)] \; ,
\end{aligned}$$

where $\Gamma_{1/2}$ are appropriate combinations of *Dirac* matrices, the p's and m's momenta and masses of particle a/b respectively. You can assume that the neutrino masses are zero.

iii. Use previous results to evaluate the traces (*Hint*: Instead of mulitplying everything out, you should try to use (a) ii. !!!).

iv. Finally plug everything together and compute $\langle|\mathcal{M}|^2\rangle$ (*Hint*: You might need $\epsilon^{\mu\nu\sigma\rho}\epsilon_{\mu\nu\kappa\tau} = -2(\delta^\sigma_\kappa\delta^\rho_\tau - \delta^\rho_\kappa\delta^\sigma_\tau)$).

It's now straight forward to derive the muon decay rate and lifetime for this process, but we will not do this here...

[1] Maybe Mark does not like this, I do! ;-)