Physikalisches Institut
Universität Bonn
Theoretische Physik

Homework 1
14.10.02
WS 02/03

Exercises on 'Elementary Particle Physics'

Prof. H. Dreiner

1. *The adjoint Dirac equation and currents*

   (a) Define $\bar{\psi} = \psi^\dagger \gamma^0$ and use the covariant form of the *Dirac* equation to derive the adjoint *Dirac* equation

   $$i\partial_\mu \bar{\psi}\gamma^\mu + m\bar{\psi} = 0 \ .$$

   (b) Show that the probability current $j^\mu \equiv \bar{\psi}\gamma^\mu\psi$ is conserved. What can you say about the probability density j^0?

2. *Completeness relations*

   In exercise 1 we have seen that the solutions $u^{(1,2)}(\vec{p})e^{-ip\cdot x}$ describe free particles (e.g. an electron) of energy E and momentum $\vec{p}$, whereas the two negative energy solutions are to be associated with the antiparticles. We want to use the so called antiparticle description, namely that an antiparticle of energy E and momentum $\vec{p}$ is descibed by a $-E, -\vec{p}$ particle solution. For convenience define

   $$u^{(3,4)}(-\vec{p})e^{-i(-p)\cdot x} \equiv v^{(2,1)}(\vec{p})e^{ip\cdot x} \ .$$

   Note that then for the antiparticle $p^0 \equiv E \geq 0$! The v's are called antiparticle (e.g. positron) spinors.

   (a) Define $\not{p} \equiv \gamma^\mu p_\mu$ (we will use this 'slash' abbreviation for any four-vectors in the future). In exercise 1 we found

   $$(\not{p} - m)u(\vec{p}) = 0 \ .$$

   What is the equivalent equation for $v(\vec{p})$?

   (b) What are the corresponding equations for $\bar{u}$ and $\bar{v}$?

   (c) Show that

   $$u^{(r)\dagger}u^{(s)} = 2E\delta_{rs} \ , \quad v^{(r)\dagger}v^{(s)} = 2E\delta_{rs} \ ,$$

   where r and s are running from 1 to 2 now of course.

   (d) Show that (no sum over (s) here)

   $$\bar{u}^{(s)}u^{(s)} = 2m \ , \quad \bar{v}^{(s)}v^{(s)} = -2m \ .$$

(e) Derive the completeness relations

$$\begin{aligned} \sum_{s=1,2} u^{(s)}(\vec{p})\, \bar{u}^{(s)}(\vec{p}) &= \not{p} + m \,, \\ \sum_{s=1,2} v^{(s)}(\vec{p})\, \bar{v}^{(s)}(\vec{p}) &= \not{p} - m \,. \end{aligned}$$