Physikalisches Institut
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Theoretische Physik

Homework 2
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WS 02/03

Exercises on 'Elementary Particle Physics'

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1. *Covariance of the Dirac equation*

   Consider the *Dirac* equation in two different frames (x and x')

   $$\begin{aligned} i\gamma^\mu \partial_\mu \psi(x) - m\psi(x) &= 0\,, \\ i\gamma^\mu \partial'_\mu \psi'(x') - m\psi'(x') &= 0 \end{aligned}$$

   and let them be related by a *Lorentz* transformation, i.e. $x'^\mu = \Lambda^\mu_{\ \nu} x^\nu$. Then there must exist a local relation $\psi'(x') = S(\Lambda)\psi(x)$, where S acts on the spinors only.

   (a) Show that covariance then requires that $S(\Lambda)\gamma^\mu S^{-1}(\Lambda) = (\Lambda^{-1})^\mu_{\ \nu}\gamma^\nu$.

   We first want to consider infinitesimal proper *Lorentz* transformations. Remember that then to linear order $\Lambda^\mu_{\ \nu} = g^\mu_{\ \nu} + \omega^\mu_{\ \nu}$ and $(\Lambda^{-1})^\mu_{\ \nu} = g^\mu_{\ \nu} - \omega^\mu_{\ \nu}$ with $\omega_{\mu\nu}$ antisymmetric.

   (b) Define $\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ and show that (again to linear order)
      (i) $S_L = 1 - \frac{i}{4}\sigma_{\mu\nu}\omega^{\mu\nu}$ does the job.
      (ii) $S_L^{-1} = \gamma^0 S_L^\dagger \gamma^0$.
      (iii) $\gamma^5 S_L = S_L \gamma^5$.

   Now consider the space inversion or parity operation $\Lambda = \mathrm{diag}(1, -1, -1, -1)$.

   (c) Show that $S_P = \gamma^0$ does the job.

2. *Bilinear covariants*

   In homework 1 we found an example for a current, namely $\bar\psi\gamma^\mu\psi$. Here we want to extend this definition to the most general form of bilinears $\bar\psi(4\times 4)\psi$ with definite properties under *Lorentz* transformations, where the 4×4 matrix is a product of γ-matrices.

   (a) Consider the following bilinears

   $$\bar\psi\psi\,, \quad \bar\psi\gamma^\mu\psi\,, \quad \bar\psi\sigma^{\mu\nu}\psi\,, \quad \bar\psi\gamma^5\gamma^\mu\psi\,, \quad \bar\psi\gamma^5\psi\,.$$

   How many independent components are there for each of them? Are there other bilinears linearly independent of those given above? Explain your answer.

(b) With help of exercise 2, 3. we now want to classify the bilinears in terms of their transformation properties under proper *Lorentz* transformations and ('intrinsic') parity. Show that the bilinears given above transform as a scalar, a vector, a tensor, an axial (or pseudo-) vector and a pseudoscalar.