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Example sheet 5
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Elementary Particle Physics II

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1. **The Minimal Supersymmetric Standard Model**
All standard model fields can be included in the following superfields:

$$\text{Quarks :} \quad Q_i = (\mathbf{3}, \mathbf{2}, -\frac{1}{6})\,, \quad U_i^R = (\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3})\,, \quad D_i^R = (\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3})\,. \tag{1}$$

$$\text{Leptons :} \quad L_i = (\mathbf{1}, \mathbf{2}, \frac{1}{2})\,, \quad E_i^R = (\mathbf{1}, \mathbf{1}, -1)\,, \quad \Big(\nu_i^R = (\mathbf{1}, \mathbf{1}, 0)\Big)\,. \tag{2}$$

$$\text{Higgs :} \quad H_u = (\mathbf{1}, \mathbf{2}, -\frac{1}{2})\,, \quad H_d = (\mathbf{1}, \mathbf{2}, \frac{1}{2})\,. \tag{3}$$

Given in the brackets are the transformation properties under $SU(3) \times SU(2) \times U(1)$. The index $i = 1, 2, 3$ labels the three different generations.

   (a) Write the most general cubic superpotential for these fields which is invariant under the Standard Model gauge group.

   (b) Identify the terms that conserve baryon and lepton number and those that do not. Verify that *R-parity* $P_R \equiv (-1)^{3(B-L)+2s}$ forbids exactly those terms that violate baryon or lepton number.

2. **The MSSM Higgs sector**

   (a) Using the R-parity preserving part of the superpotential just derived, find the part of the scalar potential that contains mass terms for $H_u = (H_u^+, H_u^0)$ and $H_d = (H_d^0, H_d^-)$.

   (b) Add to that the D-term contribution to the scalar potential coming from the gauge couplings in the Lagrangian:

$$V_{D-term} = \frac{1}{2} \sum_a g_a^2 \left(\phi^* T^a \phi\right)^2\,, \tag{4}$$

   where ϕ should be replaced with H_u and H_d respectively.

(c) Looking at the full scalar potential for the Higgs fields in unbroken SUSY, is a breaking of electroweak symmetry possible?

(d) Include the soft SUSY breaking terms

$$\mathcal{L}_{soft} = -m_{H_u}^2 \, |H_u|^2 - m_{H_d}^2 \, |H_d|^2 - (bH_uH_d + c.c.) + \cdots \tag{5}$$

in the scalar potential. We want the resulting potential's minimum to break electroweak symmetry.
(It is possible to set $\langle H_u^+ \rangle = \langle H_d^- \rangle = 0$ through an SU(2) gauge transformation. b, $\langle H_u^0 \rangle$ and $\langle H_d^0 \rangle$ can be made real and positive by a phase redefinition.)
You should now have

$$\begin{aligned} V &= \left(|\mu|^2 + m_{H_u}^2\right) \left|H_u^0\right|^2 + \left(|\mu|^2 + m_{H_d}^2\right) \left|H_d^0\right|^2 - \left(b\, H_u^0 H_d^0 + c.c.\right) \\ &\quad + \frac{1}{8} \left(g^2 + g'^2\right) \left(\left|H_u^0\right|^2 - \left|H_d^0\right|^2\right)^2 . \end{aligned} \tag{6}$$

(e) One requirement for successful electroweak symmetry breaking is a negative $(\text{mass})^2$ term for at least one linear combination of the Higgs fields. What inequality does b have to satisfy to achieve this? The second requirement is that the potential should be bounded from below. When is this guaranteed?

(f) Show that $|\mu|^2$, $m_{H_{u,d}}^2$ and b can be related through m_Z^2 if we require agreement with the experimental result for the Higgs VEV:

$$\langle v \rangle^2 = \langle H_u^0 \rangle^2 + \langle H_d^0 \rangle^2 = \frac{2\, m_Z^2}{g^2 + g'^2} \approx (174\,\text{GeV})^2 . \tag{7}$$

(Since only the sum of the squares of $\langle H_u^0 \rangle$ and $\langle H_d^0 \rangle$ is fixed experimentally, the parameter β is introduced to parameterize the remaining choice. One defines $\tan\beta \equiv \langle H_u^0 \rangle / \langle H_d^0 \rangle$.)

(g) (*) Check that the relations you found satisfy the constraints in (e).

(h) After electroweak symmetry breaking, three of the eight real scalar degrees of freedom of the two Higgs multiplets are swallowed to give mass to the Z^0 and $W^\pm$ bosons. The remaining physical fields are usually named A^0 (a neutral CP-odd pseudoscalar), $H^\pm$ (charged scalars that are conjugates to each other), H_0 and h_0 (a heavy and light CP-even scalar field).
Obtain the mass matrix for H_0 and h_0 (they are a mixture of $\Re(H_u^0) - \langle H_u^0 \rangle$ and $\Re(H_d^0) - \langle H_d^0 \rangle$). *(You can use $m_{A^0}^2 = 2b/\sin 2\beta$ to simplify the notation.)*
Show that m_{h^0} has an upper bound.