Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 1
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SS 06

Exercises on Elementary Particle Physics II

Prof. Dr. H.-P. Nilles

Exercises: Thursdays 10:15 - 12:00 Ü4 AVZ

1. *Higgs mass corrections*

   In this exercise, we calculate the corrections to the Higgs mass due to fermion- and scalar-loops. First, the relevant part of the Lagrangian reads

$$\mathcal{L}_{\text{Higgs-Top}} = -\left(f\bar{Q}_L H_c u_R + h.c.\right),$$

   where f denotes the Yukawa coupling strength. For the quark Q_L and u_R, we consider the heaviest quark, the top quark. After the Higgs gets a vev ($H_c = (\phi^0, 0)^T$ and $\phi^0 = \frac{1}{\sqrt{2}}(v+h)$), the Lagrangian reads,

$$\mathcal{L}_{\text{Higgs-Top}} = -\left(\frac{f}{\sqrt{2}}\bar{t}_L t_R(v+h) + h.c.\right),$$

   and the top-mass is defined as $m_t = \frac{fv}{\sqrt{2}}$.

   (a) Draw the Feynman diagram for the one-loop correction of the Higgs mass due to a top-loop.

   (b) Show that the amplitude for this process reads

$$\Pi_t^{hh}(0) = -6f^2 \int \frac{d^4k}{(2\pi)^4}\left(\frac{1}{k^2 - m_t^2} + \frac{2m_t^2}{(k^2 - m_t^2)^2}\right).$$

   Hint: A factor 3 comes from the three colors of a top quark and the minus-sign from the fermion in the loop.

This integral diverges quadratically. One way of solving this problem is to introduce complex scalars that couple to the Higgs, such that their contribution to the Higgs mass cancels the top contribution. For convenience, we denote these scalars $\tilde{t}_L$ and $\tilde{t}_R$ and call them s-tops. The s indicates supersymmetry. (Note that for the s-tops the subscripts L and R do not denote left- and right-chiral fields.) Like the top, both s-tops are $\mathrm{SU}(3)_c$ triplets.

(c) Compare the degrees of freedom for both, the top and the s-tops.

(d) The couplings of the s-tops to the Higgs are described by the following terms in the Lagrangian

$$\mathcal{L}_{\text{Higgs-s-top}} = \tilde{\lambda}_t |\phi^0|^2 \left(|\tilde{t}_L|^2 + |\tilde{t}_R|^2\right) + \left(f A_t \phi^0 \tilde{t}_L \tilde{t}_R^* + h.c.\right).$$

What are the vertex factors for the following couplings: $h\ h\ \tilde{t}_L\ \tilde{t}_L$, $h\ h\ \tilde{t}_R\ \tilde{t}_R$, $h\ \tilde{t}_L\ \tilde{t}_L$, $h\ \tilde{t}_R\ \tilde{t}_R$ and $h\ \tilde{t}_R\ \tilde{t}_L$?

(e) Draw all Feynman diagrams that contribute to the Higgs mass.

Hint: you should find three different types of diagrams. The diagram that involves a $h\ h\ \tilde{t}_L\ \tilde{t}_L$ vertex gets an additional factor of 2, as will be explained in the exercise.

(f) Compute the amplitude $\Pi_{\tilde{t}}^{hh}(0)$ for these diagrams.

(g) Next, assume that the s-top Yukawa coupling $\tilde{\lambda}_t$ depends on the top Yukawa coupling f in the following way: $\tilde{\lambda}_t = -f^2$. How divergent is the sum of both contributions (b) and (f) to the Higgs mass?

(h) Show that in the case that the s-top masses are equal to the top mass (and additionally $A_t = 0$) the correction vanishes. Therefore, supersymmetry can solve the hierarchy problem.