Physikalisches Institut
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Theoretische Physik

Exercises 3
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Exercises on Theoretical Particle Physics II

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In this exercise sheet we learn how to construct SUSY invariant actions. Starting with a left-chiral superfield ϕ, we can write down the most general SUSY invariant action as

$$S = \int d^4x \left[\int d^2\theta d^2\bar{\theta} K(\phi, \phi^\dagger) + \int d^2\theta W(\phi) + \text{h.c.} \right]. \tag{1}$$

The function K is the *Kähler potential* and W is the *superpotential*. The hermitian conjugate is added in order to make the expression real. W is called super*potential*, as it only contributes potential terms. The kinetic terms come from K. For the moment, think of them as aritrary functions of the superfields. The first exercise is devoted to the superpotential and the second to the Kähler potential. In the third exercise, we combine these two building blocks to get the simplest example of a SUSY invariant theory, the *Wess-Zumino model*.

3.1 The Superpotential *(5 credits)*

In this exercise we check that the superpotential is indeed SUSY invariant and investigate its structure. From exercise 1.3, we know that the expansion of a field in θ is finite. The left-chiral superfield ϕ is commonly expanded as:

$$\phi_L(x, \theta) = \varphi(x) + \sqrt{2}\theta\psi(x) + F(x)\theta\theta, \tag{2}$$

where φ and F are bosonic fields and ψ is fermionic.

(a) Check that an arbitrary function of left-chiral superfields is again a left-chiral superfield. In particular, $W(\phi)$ is a left-chiral superfield. *(1 credit)*

(b) Show that the part containing the superpotential in (1) is indeed SUSY invariant up to a total derivative (why doesn't this matter in the action?). To show this, use the infinitesimal SUSY trafo $\delta_{(\epsilon,\bar{\epsilon})} = \epsilon Q + \bar{Q}\bar{\epsilon}$, with Q and $\bar{Q}$ derived in 2.1)g) of the last exercise sheet.
Hint: Remember from exercise 1.3 how the Grassmann integration acts on a polynomial in θ. *(1 credit)*

(c) Let us take the superpotential

$$W = m\phi^2 + \lambda\phi^3. \tag{3}$$

Calculate all occuring terms in the left-chiral representation using (2). *(3 credits)*

3.2 The Kähler potential *(8 credits)*

In this exercise we look at the most simple Kähler function $K = \phi\phi^\dagger$ and investigate its structure and properties.

(a) Show that the part containing K in (1) is also SUSY invariant up to a total derivative, cf. 3.1)b). *(1 credit)*

(b) Show that $[\phi_L(x, \theta)]^\dagger$ transforms in the right-chiral representation of the SUSY algebra. Argue that $V = \phi\phi^\dagger$ is a vector superfield. *(1 credit)*

(c) Use the relations between S, S_L, and S_R derived in 2.1)f) to determine the relations between ϕ, ϕ_L, and ϕ_R. *(3 credits)*

(d) Calculate all terms coming from $K = \phi\phi^\dagger$. Use again the left-chiral representation.
Hint: $\phi\phi^\dagger$ is not chiral and transforms in a non-chiral representation. Use (c) to switch between the representations. You will also need some of the identites derived on exercise sheet 1. *(3 credits)*

3.3 The Wess-Zumino model *(8 credits)*

By combining the results of the previous two exercises we can build the Wess-Zumino model:

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\phi\phi^\dagger + \left[\int d^2\theta m\phi^2 + \lambda\phi^3 + \text{h.c.}\right]. \tag{4}$$

Let us investigate its properties:

(a) In the first two exercises, we used left-chiral fields, while the Lagrange density (4) is written in terms of non-chiral fields. Argue that the shift which connects the two representations (derived in 3.2)c)) does not change the Lagrange density. *(1 credit)*

(b) The F-field in (2) is a so-called auxiliary field, i.e. its equation of motion (EOM) is purely algebraic. Calculate its EOM and use the result to eliminate F from the Lagrange density given in (4). *(2 credits)*

(c) Show that the scalar potential $V(\phi)$ is obtained from the superpotential via

$$V(\phi) = \left|\frac{\partial W(\varphi)}{\partial\varphi}\right|^2, \tag{5}$$

where $W(\varphi)$ means to take the superpotential $W(\phi)$ and set all components but φ to zero. *(1 credit)*

(d) Calculate the equations of motions from the Lagrange density (4) for φ and ψ (leave out the interaction term). Read off the masses of the fields. What do you observe?
Hint: Decouple the EOM of ψ and $\bar{\psi}$ by taking another derivative. *(4 credits)*