Physikalisches Institut
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Theoretische Physik

Exercises 11
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Exercises on Theoretical Particle Physics II

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11.1 Extra dimensions (*20 credits*)

We will consider here effective theories in 4D from a dimensional reduction of a 5D theory. First in a simple example we will get the Kaluza-Klein tower en 4D from a 5D Scalar. Then we will show that from gravity in 5D one gets Gravity + QED in 4D. In the following, we take into account a five-dimensional theory where the fifth coordinate is curled up on a circle, $x^4 \equiv y \cong y + 2\pi R$. The index M runs from 0 to 4.

(a) Take a free complex scalar field with the action

$$S = \int d^5x \left((\partial_M \phi)^\dagger \partial^M \phi + M^2 \phi^\dagger \phi\right) \tag{1}$$

Show, using Fourier expansion, that the theory is equivalent to a four-dimensional theory with an infinite tower of fields ϕ_n with masses $m_n^2 \sim M^2 + n^2/R^2$. This is called the Kaluza-Klein tower. (*2 credits*)

(b) Now we turn to gravity(this calculation is long, so points will be given scaled to the maximum score). The action is of the usual Einstein-Hilbert form

$$S = -\frac{1}{2k_5^2} \int d^4x dy e^{(5)} R^{(5)} \tag{2}$$

Here $e^{(5)}$ and $R^{(5)}$are expressed in terms of the metric zero-mode in 5D, $G_{MN} = g^{(0)}_{MN}$.

$$G_{MN} = \phi^{-1/3} \begin{pmatrix} g_{\mu\nu} + \phi A_\mu A_\nu & \phi A_\mu \\ \phi A_\nu & \phi \end{pmatrix}, \quad G^{MN} = \phi^{1/3} \begin{pmatrix} g^{\mu\nu} & -A^\mu \\ -A^\nu & \frac{1}{\phi} + A_\mu A^\mu \end{pmatrix} \tag{3}$$

Obtain the the reduction $e^{(5)} = \sqrt{-detG_{MN}} = \phi^{-1/3} e$, where $e = \sqrt{-detg_{\mu\nu}}$. Now compute the 5D Ricci-Scalar $R^{(5)}$ in terms of the 4D fields to get

$$S = -(2\pi R) \int d^4x \frac{e}{2k_5^2} \left(R + \frac{1}{4}\phi F_{\mu\nu}F^{\mu\nu} + \frac{1}{6\phi^2}\partial^\mu \phi \partial_\mu \phi\right) \tag{4}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and R is the Ricci-scalar in 4D. You have obtained gravity in 4D + $U(1)$ gauge field and a scalar! (*15 credits*)

(c) Add now an additional scalar to the 5D theory

$$S_\rho = \int d^4x dy e^{(5)} \left(G^{MN} \partial_M \rho \partial_N \rho\right) \tag{5}$$

Expand ρ in its fourier modes

$$\rho(x,y) = \sum_{-\infty}^{\infty} \rho_k(x) e^{iky/R}, \tag{6}$$

and get the 4D action for the infinite tower of Kaluza-Klein modes. What changes with respect to the case (a)? *(3 credits)*

11.2 Weak interaction breaking induced by supergravity *(17 credits)*

We will consider a theory in which the only way the hidden and the observable sectors are coupled is via gravity Phys.Lett.120B.p.346(1982). SUSY will be broken by the hidden sector, inducing a breaking of $SU(2)\times U(1)$ in the low energy theory. Take a set of chiral multiplets in the hidden sector z_i and in the observable sector y_a. In this context the symbols are used to denote the scalar components. The form a the superpotential will be

$$W(z_i, y_a) = h(z_i) + g(y_a). \tag{7}$$

Recall the supergravity formulas

$$\begin{aligned} G &= -\frac{K(\phi_i,\phi_i^*)}{M^2} - \log\frac{|W(\phi_i)|^2}{M^6} \\ V &= -M^4 \exp(-G)(3 + G_{AB}G^{BC^*}G_{C^*}) + 1/2 D_\alpha D^\alpha \\ D_\alpha &= e_\alpha \sum_a y_a^* T^\alpha y_a. \end{aligned} \tag{8}$$

Where T^α is the gauge group generator, and e_α the associated coupling constant.

(a) Consider for the Kähler potential the canonical form $K = \phi_i\phi_i^*$. Obtain G and V in terms of the fields z_i and y_a. *(3 credits)*

(b) Consider a SUSY breaking vacuum in the hidden sector with vevs given by

$$\langle z_i\rangle = b_i M, \quad \langle h\rangle = \bar{m}M^2, \quad \langle h_i\rangle = \langle\frac{\partial h}{\partial z_i}\rangle = a_i^*\bar{m}M. \tag{9}$$

Here M is of the order of the Planck scale. Obtain the condition for the coefficients a_i and b_i such that $E_{vac} = V|_{\langle z\rangle} = 0$. *(2 credits)*

(c) Lets obtain the low energy effective potential V_{LE}. This is done by replacing z_i, h_i, h by its vevs, and keeping the terms which do not vanish in the limit $M\to\infty$. First compute the gravitino mass $m_{3/2}$, and get V_{LE} in terms of it and $y_a, \hat{g}(y_a)$ and A:

$$\begin{aligned} m_{3/2} &= Me^{-G/2} \\ \hat{g}(y_a) &= \exp(|b^i|^2/2) g(y_a) \\ A &= (a_i^* + b_i^*) b_i. \end{aligned} \tag{10}$$

Give an interpretation of the different terms in the low energy effective potential. *(4 credits)*

(d) Now consider the special case A real. Show that for $|A| < 3$ the potential V_{LE} is positive definite and has a minimum at $y_a = 0$. So for A real only theories with $|A| > 3$ can result in spontaneous breaking of $SU(2)\times U(1)$ at tree level. *(3 credits)*

Hint: Use the relation $\sum_a g_a y_a = 3g$. Compare V_{LE} with the expansion of $|\hat{g}_a \pm m_{3/2} y_a^*|^2$.

(e) Assume A real and greater than 3, choose

$$\hat{g} = \lambda Y H \bar{H} + \frac{1}{3}\sigma Y^3. \tag{11}$$

Here H and $\bar{H}$ are $SU(2)$ doublets of opposite hypercharge, and Y is an $SU(2) \times U(1)$ singlet. Study the existence of an absolute minimum with $V_{LE} < 0$ in the range of parameters

$$1 < \sigma/\lambda < \frac{1}{6}(A + (A^2 - 8)^{1/2})^2. \tag{12}$$

Where Y and the neutral components of the H and $\bar{H}$ doublets have non-vanishing vevs, and $\langle H \rangle = \langle \bar{H} \rangle^*$ in order to minimize D^2. (*5 credits*)