Physikalisches Institut
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Theoretische Physik

Exercise 7
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SS 13

Exercises on Theoretical Particle Astrophysics

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Due 7th June

7.1 Thermal distribution function of particles *7 points*

We have the following equations for the number density n, the energy density ρ and the pressure P (suppressing time dependence):

$$n = \int d^3p f(\mathbf{p}) \quad ; \quad \rho = \int d^3p E f(\mathbf{p}) \quad ; \quad P = \int d^3p \frac{|p|^3}{3E} f(\mathbf{p})$$

The particle momentum p and its energy E are related by $E^2 = p^2 + m^2$. The distribution function f is given by

$$f(\mathbf{p})d^3p = \frac{g}{(2\pi)^3} \frac{d^3p}{\exp[(E(\mathbf{p}) - \mu)/T] \pm 1}$$

(a) By integrating over the distribution function, find the expressions for n, ρ, P in terms of temperature in (i) relativisitic ($T \gg m$), nondegenerate limit ($T \gg \mu$), and (ii) nonrelativistic limit ($m \gg T$). *(4 points)*
(Hint: You will obtain different formulae for bosons and fermions in (a). The Riemann-zeta function is given as

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \,,$$

and $\zeta(3) = 1.202...$*)*

(b) The entropy density can be approximated,

$$s \equiv \frac{\rho + P - \sum_i \mu_i n_i}{T} \approx \frac{\rho + P}{T}$$

and is *always* dominated by radiation. Does it contradict with the fact that the radiation density is negligible in the current universe? *(3 points)*

7.2 Decoupling of neutrinos from the thermal bath *13 points*

At high temperatures, neutrinos are kept in thermal equilibrium with the charged leptons by the weak interactions. At a given temperature T_d (to be calculated below) the weak interactions become inefficient and the neutrinos decouple. To have a rough (but very quick!) estimate of T_d one usually compares the interaction rate Γ of the process with the expansion rate H of the universe. Use $H \sim T^2/M_{pl}$; the interaction rate is instead given by $\Gamma \sim \nu\sigma n$, where $\nu \sim c = 1$ is the (average) velocity of the neutrinos, and where the (average) cross section σ can be estimated as

$$\sigma \sim G_F^2 E^2 \,, \tag{1}$$

where $G_F \sim 10^5 \mathrm{GeV}^2$ is the Fermi constant and E is the energy exchanged in the process. The above relation (1) is valid for energies much smaller than the mass of the $W^\pm$ and Z bosons, $E \ll 80$ GeV. Finally, n is the number density of the neutrinos, $n \equiv T^\alpha$. If you do not remember α, you can easily get it by dimensional arguments, remembering that both Γ and T have dimensions of energy (use GeV in this exercise).

(a) Use H and Γ to give an estimate on T_d. Check that $T_d ll 80$ GeV, so that eg. (1) is indeed valid. *(4 points)*

(b) After the neutrinos have decoupled, the thermal bath is made only by $\gamma, e^\pm$. At a temperature $T_\gamma < T_d$, the electron /positrons annihilate, and their energy is transferred to the photons, but not to the neutrinos (since they are decoupled!). As a consequence, the temperature of the photons increases to $\tilde{T}_\gamma > T_\gamma$, while the ones of the neutrinos remain $T_\nu = T_\gamma$. Calculate $\tilde{T}_\gamma/T_\nu$. *(4 points)*

(c) In this exercise, you have assumed an instantaneous decoupling of the neutrinos at T_d . Actually, this is not precisely the case, since the decoupling is not a sudden process, but it lasts for some time (approximately, from $T \sim 5$ MeV to $T \sim 0.1$ MeV). When the $e^\pm$ annihilate, the neutrinos are not completely decoupled, and they receive some energy from the annihilation. Precise numerical calculations give $\tilde{T}_\gamma/T_\nu = 1.399$. Compare it with the analytical result found in the part (2) of this exercise. *(5 points)*