Exercises on Theoretical Particle Physics II

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10. SUSY Breaking (*8 credits*)

SUSY is broken spontaneously if $\langle Q_\alpha \rangle \neq 0$. This, in turn, is equivalent to the existence of an $|X\rangle$ such that $\langle X|Q_\alpha|0\rangle \neq 0$, or

$$\langle 0|\{Q_\alpha, \hat{X}\}|0\rangle = \langle \delta_{(\epsilon,\bar{\epsilon})}\hat{X}\rangle \neq 0, \tag{1}$$

where $\hat{X}$ is any operator in the theory and $\langle \delta_{(\epsilon,\bar{\epsilon})}\hat{X}\rangle$ denotes the VEV of the SUSY variation of the operator $\hat{X}$. We will consider the classical limit at tree level (without quantum corrections) in which $\langle \delta_{(\epsilon,\bar{\epsilon})}\hat{X}\rangle = \delta_{(\epsilon,\bar{\epsilon})}X$ for a classical field X.

(a) While SUSY should be broken, Poincaré invariance should be maintained. Which operators $\hat{X}$ can be allowed to develop a VEV $\langle \hat{X}\rangle$ without breaking the Poincaré invariance of the vacuum? What does this imply for the SUSY variation? (*2 credits*)

(b) Look at the SUSY variations of all fields in the chiral multiplet. What are the consequences of the vanishing of $\delta_{(\epsilon,\bar{\epsilon})}\psi$ for SUSY breaking and for the potential $V = |F|^2$? When is SUSY broken? (*2 credits*)

(c) Now look at the vector multiplet with component fields V^μ, λ, and D. Their SUSY variations are

$$\delta_{(\epsilon,\bar{\epsilon})}V^\mu = -\mathrm{i}\bar{\lambda}\bar{\sigma}^\mu\epsilon + \mathrm{i}\bar{\epsilon}\bar{\sigma}^\mu\lambda\,,$$
$$\delta_{(\epsilon,\bar{\epsilon})}\lambda = \bar{\sigma}^{\mu\nu}\epsilon F_{\mu\nu} + \mathrm{i}\epsilon D\,,$$
$$\delta_{(\epsilon,\bar{\epsilon})}D = -\epsilon\sigma^\mu D_\mu\bar{\lambda} - D_\mu\lambda\sigma^\mu\bar{\epsilon}\,,$$

where D_μ denotes the covariant derivative. Perform the same analysis as in (b). Note that in this case the potential is $V = \frac{1}{2}D^2$. (*2 credits*)

(d) Alternatively, we can look at the SUSY algebra itself. Express the Hamiltonian $H = P^0$ in terms of Q_α and $\bar{Q}_{\dot{\alpha}}$ and infer an inequality for the energy E on the spectrum of any SUSY theory. When is the inequality an equality? (*2 credits*)

11. F-term breaking in the O'Raifeartaigh model (*12 credits*)

In the O'Raifeartaigh model, there are three (left-)chiral superfields X,Y, and Z. Let us denote the component fields of X by (x, ψ_x, F_x) (and analogously for Y and Z). We take the easiest choice for K, such that

$$\mathcal{L}_D = K(X,Y,Z)|_{\theta^2\bar{\theta}^2} = (X^\dagger X)|_{\theta^2\bar{\theta}^2} + (Y^\dagger Y)|_{\theta^2\bar{\theta}^2} + (Z^\dagger Z)|_{\theta^2\bar{\theta}^2}\,, \tag{2}$$

where the vertical bar means restriction to the highest component. The superpotential is given by

$$W(X, Y, Z) = \lambda X(Z^2 - M^2) + gYZ, \tag{3}$$

where λ, M, and g are real parameters.

(a) Calculate the scalar potential $V(x, y, z)$, i.e.

$$V(x, y, z) = |F_x|^2 + |F_y|^2 + |F_z|^2 \quad \text{and} \quad F^*_\varphi = -\frac{\partial W(x, y, z)}{\partial \varphi} \quad \text{for} \quad \varphi = x, y, z. \tag{4}$$

(*2 credits*)

(b) Show that the VEVs of F_x, F_y, and F_z in general cannot vanish simultaneously. Hence the O'Raifeartaigh model implements F-term SUSY breaking.(*2 credits*)

(c) Check that the minimum of the potential $V(x, y, z)$ is at $y = z = 0$ when $M^2 < \frac{g^2}{2\lambda^2}$. (*3 credits*)

(d) Calculate the masses of the scalars. To do so, expand the fields in terms of fluctuations around their background value defined by their VEVs (e.g. $x \to \langle x \rangle + x$). Insert the expansion into the potential and extract the terms quadratic in the fields. In order to diagonalize the mass matrix for z, use the ansatz $z = \frac{1}{\sqrt{2}}(a + ib)$. (*3 credits*)

(e) Calculate the masses of the fermions. To do this, combine ψ_y and ψ_z into a Dirac fermion ψ_D:

$$\psi_D = \begin{pmatrix} \psi_y \\ \bar{\psi}_z \end{pmatrix}.$$

As the VEV of x is undetermined, the term $x\psi_z\psi_z$ does not constitute a mass term. (*2 credits*)