Physikalisches Institut
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Theoretische Physik

Exercises 6
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Exercises on Theoretical Particle Physics II

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12. The MSSM Higgs sector (*20 credits*)

(a) Take the MSSM Superpotential,

$$\mathcal{W} = \mu H\bar{H} + y_{\mathrm{E}} L H \bar{E} + y_{\mathrm{D}} Q H \bar{D} + y_{\mathrm{U}} Q \bar{H} \bar{U}\,,$$

and calculate the F-term contributions V_F to the Higgs scalar potential in terms of the Higgs scalars

$$H|_{\theta=0} = h = \begin{pmatrix} h^0 \\ h^- \end{pmatrix}, \qquad \bar{H}|_{\theta=0} = \bar{h} = \begin{pmatrix} \bar{h}^+ \\ \bar{h}^0 \end{pmatrix}.$$

(*2 credits*)

(b) Calculate the contribution to the Higgs scalar potential which originates from D-terms V_D of the electroweak gauge multiplets. You can obtain it from the Kähler potential which reads

$$\mathcal{K} \supset \bar{H}^\dagger e^V \bar{H} + H^\dagger e^V H$$

with $V = g_1 Y V_1 + 2 g_2 T^a V_2^a$, Y being the Hypercharge, T^a the $SU(2)_L$ generators. V_1 and V_2^a are vector superfields. Take further $Y = -1$ for H and $Y = 1$ for $\bar{H}$.

(*4 credits*)

(c) Show that at this stage electroweak symmetry breaking is not possible.

(*2 credits*)

(d) Therefore we include a soft SUSY breaking sector

$$\mathcal{L}_{\text{soft}} = -V_{\text{soft}} = -m^2_{\text{soft},1}|h|^2 - m^2_{\text{soft},2}|\bar{h}|^2 - m^2_{\text{soft},3}\left(\bar{h}h + \text{h.c.}\right)\,,$$

where $|h|^2 = h^\dagger h$ and $\bar{h}h = \epsilon^{ab}\bar{h}^a h^b$. We can use an SU(2) rotation to set $\langle h^- \rangle = 0$. Show that being in a minimum is possible with $\langle \bar{h}^+ \rangle = 0$ so that electromagnetism is restored. Show that using further phase rotations we can make $m^2_{\text{soft},3}$, $\langle h^0 \rangle$ and $\langle \bar{h}^0 \rangle$ real. Combine your result from part (a) and (b) with V_{soft} to the complete Higgs scalar potential

$$V = V_F + V_D + V_{\text{soft}}.$$

At the end the potential should read

$$V(h,\bar{h}) = m_1^2|h|^2 + m_2^2|\bar{h}|^2 + m_3^2\left(\bar{h}h + \text{h.c.}\right) + \frac{g_1^2+g_2^2}{8}\left(|h|^2 - |\bar{h}|^2\right)^2 + \frac{g_2^2}{2}|h^\dagger \bar{h}|^2\,. \tag{1}$$

Identify the mass parameters m_1^2, m_2^2 and m_3^2.

(*3 credits*)

(e) To obtain electroweak symmetry breaking we require the potential to be bounded from below and that the point $h^0 = \bar{h}^0 = 0$ is not a minimum. Show that this leads to the requirements

$$2m_3^2 < m_1^2 + m_2^2\,,$$
$$m_3^4 > m_1^2 m_2^2\,.$$

(*3 credits*)

(f) After electroweak symmetry breaking, three of the eight real scalar degrees of freedom of the two Higgs multiplets are swallowed to give mass to the Z^0 and $W^\pm$ bosons. The remaining physical fields are usually named A^0 (a neutral CP-odd pseudoscalar), $H^\pm$ (two charged scalars that are conjugates to each other), H_0 and h_0 (a heavy and a light CP-even scalar field).
Take the usual convention $\langle h^- \rangle = \langle \bar{h}^+ \rangle = 0$ and calculate the 2×2 mass matrix for the fields h^- and $\bar{h}^+$. Use the potential from equation (1) and interpret your result.

(*4 credits*)

(g) Repeat the analysis from part (f) and calculate the 2×2 mass matrix for the fields $\mathrm{Im}(h^0)$ and $\mathrm{Im}(\bar{h}^0)$. Remember that $\langle h^0 \rangle$ and $\langle \bar{h}^0 \rangle$ can be taken to be real. Interpret again your result.

(*2 credits*)