String Theory

Winter Term 2008/2009

Blatt 1

Discussion: October 22, 14:00 in Hörsaal 118, AVZ

1. For any matrix A, the exponential e^A is defined by the power series,

$$e^A = \sum_{n=0}^{\infty} \frac{1}{n!} A^n \,.$$

   (a) Check the obvious properties $e^{A^T} = \left(e^A\right)^T$, $e^{A^*} = \left(e^A\right)^*$ and $e^{aA}e^{bA} = e^{(a+b)A}$ for numbers a and b!

   (b) Check that in general, $e^A e^B \neq e^{A+B}$! Under which condition does this hold? Derive the first term in the Baker–Campbell–Hausdorff formula $e^A e^B = e^{A+B+\cdots}$!

   (c) Show that

$$\det e^A = e^{\operatorname{tr} A} \,.$$

   This is a formula you should remember for life! (It is often phrased as $\ln \det = \operatorname{tr} \ln$. For which matrices is the logarithm defined?)

   (d) Show that under a variation $A \to A + \delta A$, the determinant of A varies as

$$\delta(\det A) = \det A \; \operatorname{tr}\left(A^{-1} \delta A\right) \,.$$

   (assume that A is invertible).

2. The action for a massive point particle with coordinates $x^\mu(\tau)$ is

$$S = -m \int \mathrm{d}\tau \, \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} \,.$$

   (a) Show that the action is invariant under worldline reparametrisations $\tau \to \tau'(\tau)$ and spacetime reparametrisations $x^\mu \to x^{\mu\prime}(x)$!

   (b) Assume that the time parameter is affine, i.e. $g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu \equiv \dot{x}^2 = \text{const.}$. Derive the geodesic equation as the equation of motion!

3. For particles, there also is a Polyakov-type action: Introduce an auxiliary metric h on the worldline, such that $\mathrm{d}s^2 = h_{\tau\tau} \mathrm{d}\tau^2$. The new action is

$$\widetilde{S} = \frac{1}{2} \int \mathrm{d}\tau \, \sqrt{h_{\tau\tau}} \left(h_{\tau\tau}^{-1} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - m \right)$$

   (a) Show that $\widetilde{S}$ is again invariant under reparametrisations of the worldline!

   (b) Assume $m \neq 0$. Derive the "Nambu–Goto" action S from $\widetilde{S}$!

   (c) Show that massless particles move on lightlike geodesics!