Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 5
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WS 10/11

Exercises on Theoretical Particle Physics

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–Home Exercises–
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H 5.1 Vector- and Axial Gauge Couplings *5 points*

We consider the Fermion kinetic terms in the standard model Lagrangian, see last sheet. We want to express those terms using Dirac spinors. Therefore we write

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = P_L \begin{pmatrix} \Psi_\nu \\ \Psi_e \end{pmatrix} , \qquad R = e_R = P_R \Psi_e \, ,$$

using the chiral projection operators $P_{L/R} = \frac{1}{2}\left(1 \mp \gamma^5\right)$. Furthermore, we use the covariant derivative formulated in terms of the propagating gauge fields $W_\mu^\pm, Z_\mu$ and A_μ, see last sheet.

(a) Rewrite the kinetic terms

$$\mathscr{L}_{\text{Lept}} = \overline{R}(\mathrm{i}\gamma^\mu D_\mu)R + \overline{L}(\mathrm{i}\gamma^\mu D_\mu)L$$

according to the replacements done above. At the end you should find

$$\begin{aligned} \mathscr{L}_{\text{Ferm}} = &\ \overline{\Psi}_e \left(\mathrm{i}\gamma^\mu \partial_\mu\right) \Psi_e + \overline{\Psi}_\nu \left(\mathrm{i}\gamma^\mu \partial_\mu\right) P_L \Psi_\nu \\ &+ e A_\mu \overline{\Psi}_e \gamma^\mu \Psi_e \\ &+ \frac{g}{\sqrt{2}} W_\mu^+ \overline{\Psi}_\nu \gamma^\mu P_L \Psi_e + \text{h.c.} \\ &+ Z_\mu \Big(\overline{\Psi}_e \gamma^\mu \left(c_V^e + c_A^e \gamma^5\right) \Psi_e + \overline{\Psi}_\nu \gamma^\mu \left(c_V^\nu + c_A^\nu \gamma^5\right) \Psi_\nu \Big) \, . \end{aligned} \tag{1}$$

We learn that the photon couples non-chirally to the electron, the $W^\pm$ bosons couple between the left-chiral electron and neutrino whereas the Z couples to a mixture of the vector- and the axial current. Determine the respective coupling constants c_V^i and c_A^i for $i = e, \nu$. (*2 points*)

(b) Draw the interaction vertices and give the vertex factors. (*1 point*)

(c) Now we introduce quark fields.

	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$D = d_R$	$U = u_R$
Hypercharge Y	$1/3$	$-2/3$	$4/3$
$\mathrm{SU}(2)_L$ rep.	**2**	**1**	**1**
Lorentz rep.	$(1/2, 0)$	$(0, 1/2)$	$(0, 1/2)$

Perform the above analysis for the quarks using the quark kinetic terms

$$\mathscr{L}_{\text{Quark}} = \overline{Q}(\mathrm{i}\gamma^\mu D_\mu)Q + \overline{D}(\mathrm{i}\gamma^\mu D_\mu)D + \overline{U}(\mathrm{i}\gamma^\mu D_\mu)U\,.$$

What are the electric charges of the up and down quarks? (*2 points*)

H 5.2 Gauge Boson Interactions *2 points*

The gauge boson kinetic terms contain interaction terms in the non Abelian case. In the standard model this happens for the $\mathrm{SU}(2)_L$ gauge bosons. Write these terms in the propagating fields $W^\pm_\mu$, Z_μ and A_μ and identify the interaction vertices.

H 5.3 Fermion Mass Eigenstates *8 points*

On the previous exercise sheet we have investigated the Standard Model (SM) Higgs effect on the gauge bosons. We have seen that the gauge bosons of the unbroken subgroup remain massless while those of the broken subgroup become massive. Since the SM fermions are chiral we can not write down a gauge invariant mass term. Therefore we have to employ the Higgs mechanism to generate fermion masses. Here we assume a number of N generations, i.e. we deal with fields L_i and R_i with $i = 1, \ldots, N$.

(a) The electron Yukawa couplings read

$$\mathscr{L}_{\text{Lept,Yuk}} = -G_e^{ij}\overline{L}_i\Phi R_j - \text{h.c.}\,,$$

where G_e^{ij} are some general $N \times N$ matrices called *Yukawa matrices*. By a biunitary transformation $e_R^i \to V_e^{ij} e_R^j$ and $e_L^i \to U_e^{ij} e_L^j$ one can diagonalize the Yukawa matrices. Show that after spontaneous symmetry breaking the masses for the electrons are $m_e^i = \frac{\lambda_e^i v}{\sqrt{2}}$ where λ_e^i are the eigenvalues. (*1 point*)

(b) Perform the transformation on the interaction terms in (1). The index structure is here diagonal, i.e. the family indices are contracted with a Kronecker delta. Show that the interactions with A_μ and Z_μ stay diagonal. Show that with a proper transformation of the neutrinos also the $W^\pm_\mu$ terms stay diagonal. Why is this transformation on the neutrinos allowed? This shows us that within the standard model there is no mixing between the leptons. (*1.5 points*)

(c) Now we want to consider quark masses. We can immideately write down analogous Yukawa terms for the down quarks,

$$\mathscr{L}_{\text{Down,Yuk}} = -G_d^{ij}\overline{Q}_i\Phi D_j - \text{h.c.}\,.$$

Show that they are gauge invariant. *(0.5 points)*

(d) In order to write down an up quark Yukawa term we take a closer look at $\widetilde{\Phi} := \mathrm{i}\sigma_2\Phi^*$. Show that $\widetilde{\Phi}$ transforms as a **2** of $\mathrm{SU}(2)_L$. What is the hypercharge of $\widetilde{\Phi}$? *Hint: You may first want to show that* $\sigma^2\sigma^i\sigma^2 = -\sigma^{i*}$ *and then write* $U \in \mathrm{SU}(2)_L$ *as* $U = \exp\{i\vec{a}\cdot\vec{\sigma}\}$. *(2 points)*

(e) Show that

$$\mathscr{L}_{\text{Up,Yuk}} = -G_u^{ij}\overline{Q}_i\widetilde{\Phi}U_j - \text{h.c.}\,.$$

is gauge invariant. Show that inserting the Higgs VEV results in mass matrices for the up quarks. *(1 point)*

(f) Again we perform biunitary transformations on the left- and right-handed up- and down quarks to diagonalize G_d and G_u. Then we insert the mass eigenstates into the gauge interactions. Show that the couplings to A_μ and Z_μ stay diagonal. This fact is known as the absence of flavour changing neutral currents (FCNC). *(1 point)*

(g) Now we investivate the charged currents. Show that the couplings to the $W_\mu^\pm$ now are mixed by a generally non-diagonal matrix, called the *Cabibbo–Kobayashi–Maskawa* matrix V_{CKM}. Identify this matrix in terms of the unitary transformation matrices. *(1 point)*