Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercises 6
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WS 2016/17

Exercises on Theoretical Particle Physics I

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10. Electron-Muon scattering (*10 credits*)

(a) Derive the completeness relations for Dirac particles

$$\sum_s u^{(s)}(p)\overline{u}^{(s)}(p) = \not{p} + m, \qquad \sum_s v^{(s)}(p)\overline{v}^{(s)}(p) = \not{p} - m.$$

(*1 credit*)

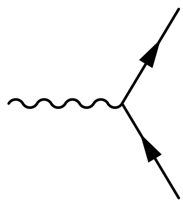
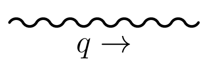
(b) The Feynman rules to calculate the amplitude $-i\mathcal{M}$ in QED are

(i) An arrow in the direction of time denotes a particle, an arrow in the opposite direction denotes an antiparticle. Assign a label i to each external particle. Assign momenta to each particle (including the internal lines) and indicate them by momentum-arrows beside the particle lines.

(ii) For the following rules, proceed "backwards" with respect to the particle arrow for each fermion line. For a particle, proceeding backwards means "opposite to the direction of time". For an antiparticle, proceeding backwards means "in the direction of time".

(iii) Write a factor $u(p_i)$ $(v(p_i))$ for every external (anti-)particle line which arrow points towards a vertex and $\overline{u}(p_i)$ $(\overline{v}(p_i))$ for lines that point away from the vertex.

(iv) The contribution from vertices and internal lines (propagators) is given by

μ  $= ie\gamma^\mu$, μ 

ν $= -i\frac{\eta_{\mu\nu}}{q^2}$.

The indices of the γ's are contracted with the $\eta_{\mu\nu}$ of the photon proparator.

(v) Use 4-momentum conservation at the vertices to eliminate the internal momenta.

Draw the Feynman graph for the process $e^-\mu^- \to e^-\mu^-$ and label the graph according to the presented rules.

(*1 credit*)

(c) Use the Feynman rules to derive the scattering amplitude as

$$\mathcal{M} = -\frac{e^2}{(p_1 - p_3)^2}\Big[\overline{u}(p_3)\gamma^\mu u(p_1)\Big]\Big[\overline{u}(p_4)\gamma_\mu u(p_2)\Big].$$

(*1 credit*)

(d) To calculate the cross section, we need to know $|\mathcal{M}|^2$. Show that

$$|\mathcal{M}|^2 = \frac{e^4}{(p_1 - p_3)^4}\Big[\overline{u}(p_3)\gamma^\mu u(p_1)\overline{u}(p_1)\gamma^\nu u(p_3)\Big]\Big[\overline{u}(p_4)\gamma_\mu u(p_2)\overline{u}(p_2)\gamma_\nu u(p_4)\Big].$$

(*1 credit*)

(e) Use part (a) to show that

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = e^4\frac{\text{tr}\Big[\Big(\not{p}_3 + m_e\Big)\gamma^\mu\Big(\not{p}_1 + m_e\Big)\gamma^\nu\Big]\,\text{tr}\Big[\Big(\not{p}_4 + m_\mu\Big)\gamma_\mu\Big(\not{p}_2 + m_\mu\Big)\gamma_\nu\Big]}{4\,(p_1 - p_3)^4}$$

where one has averaged over the initial spins and summed over the final spins.

(*2 credits*)

(f) Use the results from exercise 6 to show that

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2$$
$$= 8e^4\frac{(p_1\cdot p_2)\,(p_3\cdot p_4) + (p_1\cdot p_4)\,(p_3\cdot p_2) - (p_1\cdot p_3)\,m_\mu^2 - (p_2\cdot p_4)\,m_e^2 + 2m_\mu^2 m_e^2}{(p_1 - p_3)^4}.$$

(*2 credits*)

(g) Consider the rest frame of the muon and use $m_\mu \gg m_e$. Show that

$$(p_1 - p_3)^2 = -4p^2\sin^2\frac{\theta}{2}\,, \qquad p_1\cdot p_3 = m_e^2 + 2p^2\sin^2\frac{\theta}{2}$$
$$(p_1\cdot p_2)\,(p_3\cdot p_4) = E^2 m_\mu^2\,, \qquad p_2\cdot p_4 = m_\mu^2$$

where p labels the absolute value of the initial electron momentum, E its energy and θ is the angle between the ingoing and outgoing electron.

(*1 credit*)

(h) Use part (f) and part (g) to show that the differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 m_\mu^2}\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{1}{64\pi^2}\frac{e^4}{p^4\sin^4\frac{\theta}{2}}\left[m_e^2 + p^2\cos^2\frac{\theta}{2}\right].$$

(*1 credit*)

11. Electron-Positron annihilation Part I *(10 credits)*

(a) Draw the Feynman graph for the process $e^- e^+ \to \mu^- \mu^+$ and label the graph similar to exercise 10.

(1 credit)

(b) Use the Feynman rules to derive the annihilation amplitude as

$$\mathcal{M} = -\frac{e^2}{(p_1 + p_2)^2} \Big[\overline{v}(p_2)\gamma^\mu u(p_1)\Big] \Big[\overline{u}(p_4)\gamma_\mu v(p_3)\Big].$$

(1 credit)

(c) Derive

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2$$
$$= 8e^4 \frac{(p_1 \cdot p_4)(p_2 \cdot p_3) + (p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_2)\, m_\mu^2 + (p_3 \cdot p_4)\, m_e^2 + 2m_\mu^2 m_e^2}{(p_1 + p_2)^4}.$$

(8 credits)