Exercises on Theoretical Particle Physics I

Prof. Dr. H.P. Nilles

Due 5.12.2016

12. Electron-Positron annihilation Part II *(4 credits)*

(a) Consider the kinematic in the center-of-mass frame, use $m_\mu \gg m_e$ and show that the result of exercise 11 can be rewritten as

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = e^4 \left(1 + \frac{m_\mu^2}{E^2} + \left(1 - \frac{m_\mu^2}{E^2}\right)\cos^2\theta\right)$$

if E is the energy of the incoming electron and θ the angle between the incoming electron and outgoing muon.

(1 credit)

(b) The differential cross section for a process of two incoming and two outgoing particles can be derived using

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{s}\frac{|\boldsymbol{p_3}|}{|\boldsymbol{p_1}|}\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2$$

with $s = (p_1 + p_2)^2$. Use part (a) to derive the differential cross section.

(1 credit)

(c) Derive the total cross section σ. What would be the result if $\mathcal{M}$ is energy independent?

(2 credits)

13. Electron-Hadron scattering *(16 credits)*

(a) Use the Dirac equation to derive the Gordon decomposition

$$\overline{u}(p_1)\gamma^\mu u(p_2) = \overline{u}(p_1)\left(\frac{(p_1+p_2)^\mu}{2m} + i\frac{(p_1-p_2)_\nu}{m}\Sigma^{\mu\nu}\right)u(p_2).$$

(2 credits)

(b) We want to consider the scattering at a spin 1/2 hadron with mass M and charge e_H. To take care of the inner structure of the hadron we generalize

$$\overline{u}(p_1)\gamma_\mu u(p_2) \to \overline{u}(p_1)\Gamma_\mu(p_1,p_2)u(p_2).$$

Show that for a parity invariant force the most general ansatz is

$$\Gamma_\mu(p_1,p_2) = \gamma_\mu A + (p_1+p_2)_\mu B + (p_1-p_2)_\mu C.$$

Use the conservation of the current

$$q^\mu \overline{u}(p_1)\Gamma_\mu(p_1,p_2)u(p_2) = 0, \qquad q^\mu = (p_1 - p_2)^\mu$$

to determine C. Use then part (a) to derive

$$\Gamma_\mu(p_1,p_2) = \gamma_\mu F_1(q^2) + \frac{i\Sigma_{\mu\nu}q^\nu}{M} F_2(q^2)$$

with the so-called form factors $F_1(q^2)$ and $F_2(q^2)$.

(2 credits)

(c) Write down the Feynman graph for the scattering of an electron at a spin $1/2$ hadron in QED. According to part (b) use $ie_H\Gamma^\mu(p_3,p_4)$ at the hadron vertex instead of $ie\gamma^\mu$.

(1 credit)

(d) Show that the trace over the hadron current may be written as

$$\mathrm{tr}\left[(\not{p}_3 + M)\left(\gamma_\mu\left(F_1(q^2)+F_2(q^2)\right) - \frac{F_2(q^2)}{2M}(p_3+p_4)_\mu\right)\right.$$
$$\left.(\not{p}_4 + M)\left(\gamma_\nu\left(F_1(q^2)+F_2(q^2)\right) - \frac{F_2(q^2)}{2M}(p_3+p_4)_\nu\right)\right].$$

(1 credit)

(e) Assume that the hadrons as well as the electrons are not polarized and use further $E \gg m_e$ which means one can neglect the electron mass. E labels the electron energy. Consider further the rest frame of the hadron where θ is defined like in exercise 10. Show that the differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{e^2}{2\pi}\frac{e_H^2}{2\pi}\frac{1}{16E^2\sin^4\frac{\theta}{2}\left(1+\frac{2E}{M}\sin^2\frac{\theta}{2}\right)}$$
$$\left(\left(F_1^2(q^2) - \frac{q^2}{4M^2}F_2^2(q^2)\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\left(F_1(q^2)+F_2(q^2)\right)^2\sin^2\frac{\theta}{2}\right).$$

This result is known as the Rosenbluth formula.

(10 credits)