Abundance of cold relics in non-standard cosmological scenarios

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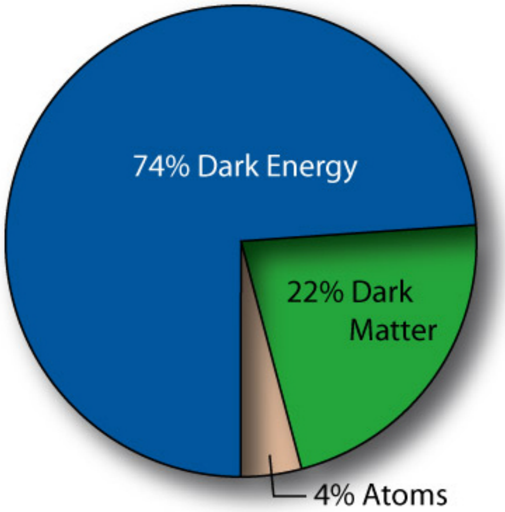
Refs:

- PRD73 (2006) 123502 [hep-ph/0603165]
- arXiv:0704.1590 [hep-ph]

1. Motivation



[http://map.gsfc.nasa.gov]

- Observations of
  - cosmic microwave background
  - large-scale structure of the universe
  - etc.

➡ Non-baryonic cold dark matter (CDM): $0.8 < \Omega_{\rm CDM} h^2 < 0.12$ (95% CL)

- Neutral, stable (long-lived) weakly interacting massive particles (WIMPs) χ are good candidates for CDM
  - Neutralino (LSP); 1st KK mode of the B boson (LKP); etc.

When WIMPs were in full thermal eq., the relic abundance naturally falls around the observed CDM abundacne: $\Omega_{\chi,\rm standard} h^2 \sim 0.1$

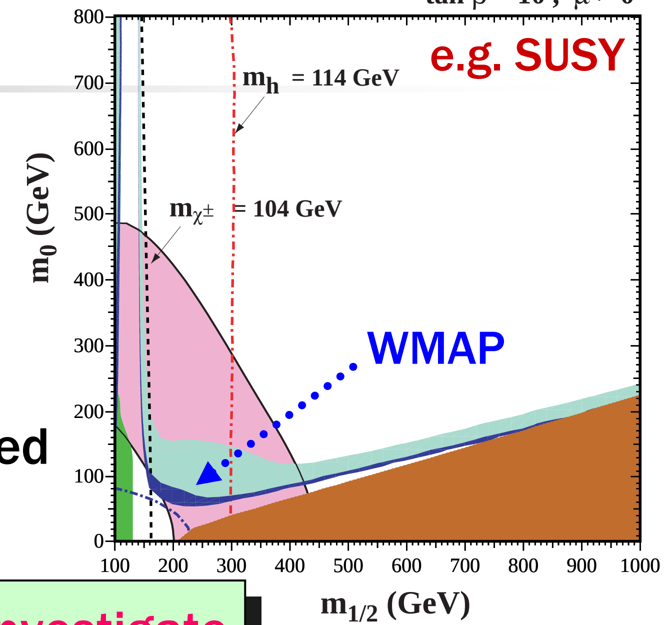
Investigation of early universe using CDM abundance

- The relic abundance of thermal WIMPs is determined by the Boltzmann equation:

$$\dot{n}_\chi + 3Hn_\chi = -\langle\sigma_{\rm eff}v\rangle(n_\chi^2 - n_{\chi,{\rm eq}}^2)$$

and the maximal temperature T_0 of RD epoch

- The (effective) cross section $\sigma_{\rm eff}$ can be determined from collider and DM detection experiments



[From Ellis et al., PLB565 (2003) 176]

We can test the standard CDM scenario and investigate the conditions of very early universe: $T_0, H, \cdots$

- Standard scenario:
  - χ was in chemical eq.
    $\Omega_\chi h^2$ is independent of T_0
  - $H = \dfrac{\pi T^2}{M_{\rm Pl}}\sqrt{\dfrac{g_*}{90}}$ (g_*: Rel. dof)

- Non-standard scenarios:
  - Low reheat temperature
  - Entropy production
  - Modified Hubble parameter
  - Non-thermal production

[Scherrer et al., PRD(1985); Salati,PLB(2003); Fernengo et al., PRD(2003); Chung et al., PRD (1999); ...]

Outline

This work

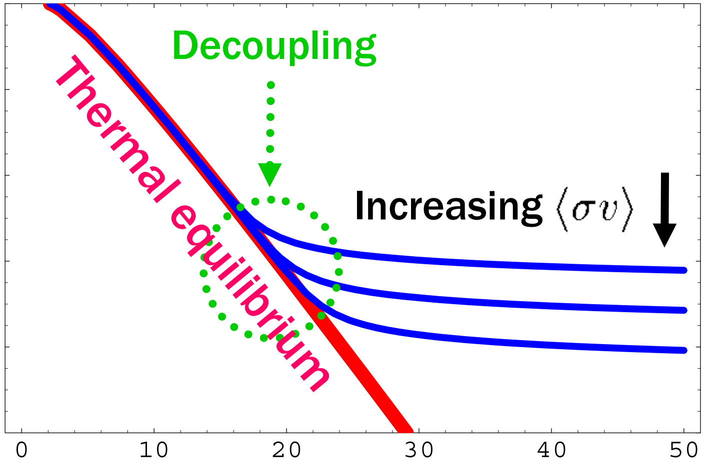
- We provide an approximate analytic treatment that is applicable to low-maximal-temperature scenarios
- Based on the assumption of CDM = thermal WIMP
  - we derive the lower bound on the maximal temperature of RD epoch
  - we constrain possible modifications of the Hubble parameter

1. Motivation
2. Standard calculation of WIMP relic abundance
3. Low-temperature scenario
4. Constrains on the very early universe from WIMP dark matter
5. Summary

2. Standard calculation of the WIMP relic abundance

[Scherrer, Turner, PRD33(1986)]

- Conventional assumptions for χ :
  - $\chi = \bar{\chi}$, single production of χ is forbidden
  - Thermal equilibrium was maintained



- For adiabatic expansion the Boltzmann eq. is

$$\frac{dY_\chi}{dx} = -\frac{\langle \sigma v \rangle s}{Hx}(Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2), \quad Y_{\chi(,\mathrm{eq})} = \frac{n_{\chi(,\mathrm{eq})}}{s}, x = \frac{m_\chi}{T} \qquad x = m/T$$

- During the RD epoch, χ and decoupled when they were non-relativistic:

$$\langle \sigma v \rangle = a + 6b/x + \mathcal{O}(1/x^2), \qquad n_{\chi,\mathrm{eq}} = g_\chi \ (m_\chi T/2\pi)^{3/2} \mathrm{e}^{-m_\chi/T}$$

$$\Rightarrow \quad \Omega_{\chi,\mathrm{standard}} h^2 \simeq 0.1 \times \left(\frac{a + 3b/x_F}{10^{-9}\ \mathrm{GeV}^{-2}} \right)^{-1} \left(\frac{x_F}{22} \right) \left(\frac{g_*}{90} \right)^{-1/2} \sim \Omega_{\mathrm{CDM}} h^2$$

3. Low-temperature scenario

- T_0 : The maixmal temperature of the RD epoch

  The initial abundance is assumed to be negligible: $Y_\chi(x_0) = 0$

- Zeroth order approximation:

  $T_0 < T_F$ ⇨ χ annihilation is negligible:

$$\frac{dY_0}{dx} = 0.028\ g_\chi^2 g_*^{-3/2} m_\chi M_{\rm Pl} e^{-2x} x \left(a + \frac{6b}{x} \right)$$

⇨ The solution is proportional to the cross section:

At late times,

$$Y_0(x \gg x_0) \simeq 0.014\ g_\chi^2 g_*^{-3/2} m_\chi M_{\rm Pl} e^{-2x_0} x_0 \left(a + \frac{6b}{x_0} \right)$$

This solution should be smoothly connected to the standard result

First order approximation

- Add a correction term describing annihilation to Y_0 : $Y_1 = Y_0 + \delta \ (\delta < 0)$
- As long as $|\delta| \ll Y_0$, the evolution equation for δ is

$$\frac{d\delta}{dx} = -1.3\ \sqrt{g_*} m_\chi M_{\rm PL} \left(a + \frac{6b}{x} \right) \frac{Y_0(x)^2}{x^2}$$

➡ The solution is proportional to σ^3

At late times,

$$\delta(x \gg x_0) \simeq -2.5 \times 10^{-4}\ g_\chi^4 g_*^{-5/2} m^3 M_{\rm Pl}^3 {\rm e}^{-4x_0} x_0 \left(a + \frac{3b}{x_0} \right) \left(a + \frac{6b}{x_0} \right)^2$$

- $|\delta|$ soon dominates over Y_0 for not very small cross section

➡ Y_1 fails to track the exact solution

Re-summed ansatz

- It is noticed that $Y_0 \propto \sigma > 0,\ \delta \propto \sigma^3 < 0$
  For large cross section, $Y_\chi(x \to \infty)$ should be $\propto 1/\langle \sigma v \rangle$

- This observation suggests the re-summed ansats:

$$Y(x) = Y_0 + \delta = Y_0\left(1 + \frac{\delta}{Y_0}\right) \simeq \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}$$

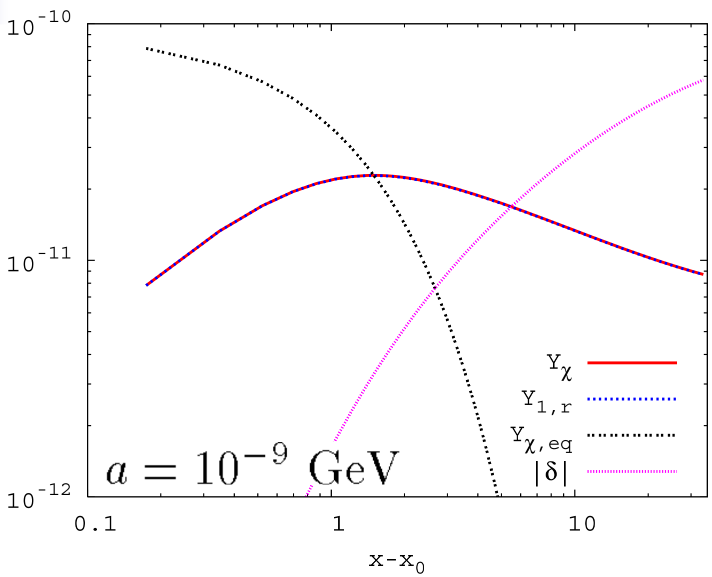
- For $|\delta| \gg Y_0$, $Y_{1,r}(x) \simeq -\frac{Y_0^2}{\delta} \propto \frac{1}{\sigma}$

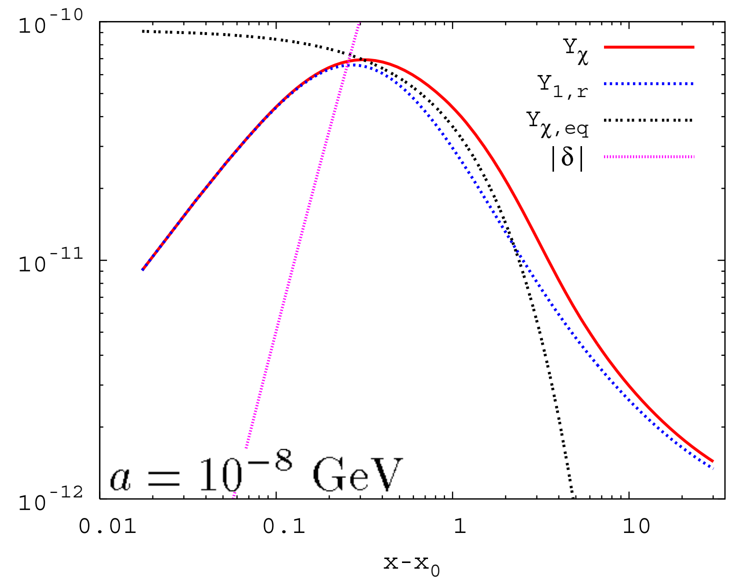
  $x_0 \to x_F$ ⇨ Standard formula

  At late times, $Y_{1,r}(x \to \infty) = \dfrac{x_0}{1.3\sqrt{g_*}m_\chi M_{\rm Pl}(a + 3b/x_0)}$

- In the case where χ production is negligible
  but the initial abundance is sizable, $Y_{1,r}$ is exact

Evolution of solutions





Y_χ : Exact result, $Y_{1,r}$: Re-summed ansatz, $b = 0,\ Y_\chi(x_0 = 22) = 0$

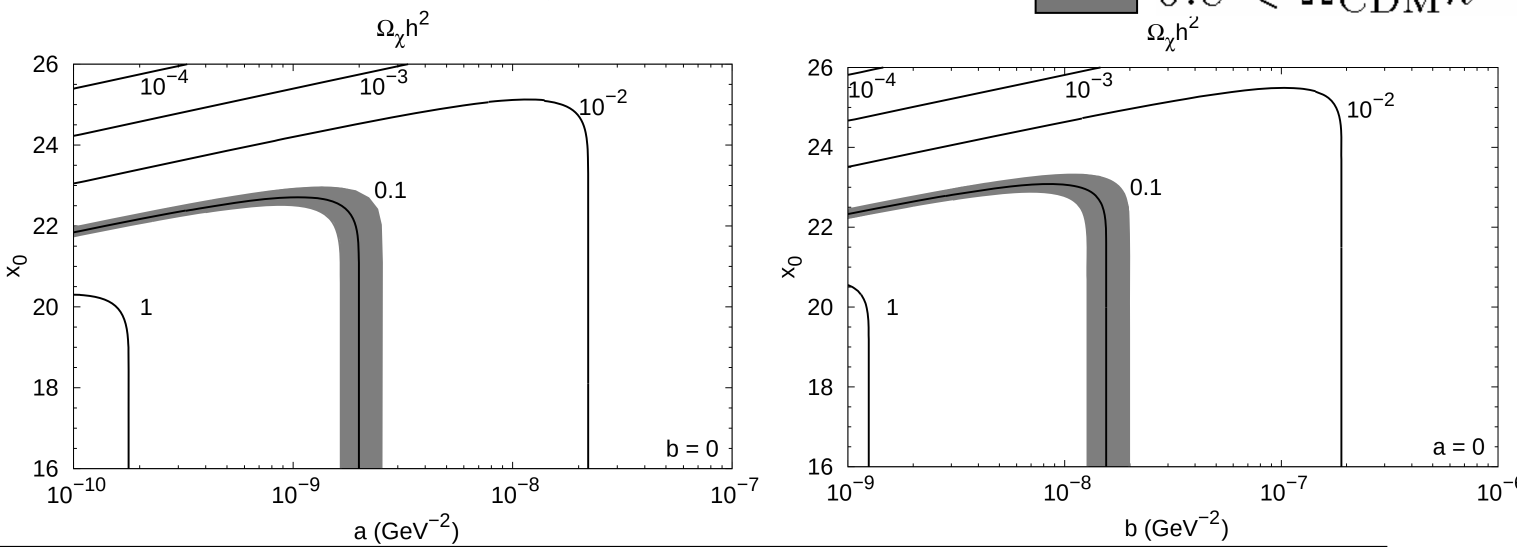
- The re-summed ansatz $Y_{1,r}$ describes the full temperature dependence of the abundance when equilibrium is not reached
- For larger cross section the deviation becomes sizable for $x - x_0 \sim 1$, but the deviation becomes smaller for $x \gg x_0$

4. Constrains on the very early universe from WIMP DM

- **Out-of-equilibrium case:** $\sigma \nearrow \Rightarrow \Omega h^2 \nearrow$; $T_0 = m_\chi / x_0 \nearrow \Rightarrow \Omega h^2 \nearrow$

  Equilibrium case: $\sigma \nearrow \Rightarrow \Omega h^2 \searrow$; Ωh^2 Independent of T_0

- **Thermal relic abundance in the RD universe:** ▇ $0.8 < \Omega_{\rm CDM} h^2 < 0.12$



Assumption that $\Omega_{\rm CDM} h^2 = \Omega_{\chi,{\rm thermal}} h^2$,

$\Rightarrow$ Lower bound on the maximal temperature: $T_0 > m_\chi / 23$

Modified expansion rate

- Various cosmological models predict a non-standard early expansion

  ⇒ Predicted WIMP relic abundances are also changed

- When WIMPs were in full thermal equilibrium, in terms of the modification parameter $A(x) = H_{\rm st}(x)/H(x)$ the relic abundance is

$$\Omega_\chi h^2 = 0.1 \left(\frac{I(x_F)}{8.5 \times 10^{-10}\ \mathrm{GeV}^{-2}} \right)^{-1}$$

$$I(x_F) = \int_{x_F}^{\infty} dx \frac{\sqrt{g_*}\langle \sigma v \rangle A(x)}{x^2}, \quad x_F = \ln \left[\sqrt{\frac{45}{\pi^5}} \xi m_\chi M_{\rm Pl} g_\chi \frac{\langle \sigma v \rangle A(x)}{\sqrt{x g_*}} \right] \Bigg|_{x=x_F}$$

If $A(x) = 1$, $x_F = x_{F,\rm st}$ and we recover the standard formula

This formula is capable of predicting the final relic density correctly

Constrains on modifications of the Hubble parameter

- In terms of $z \equiv \frac{T}{m_\chi} = \frac{1}{x}$, we need to know $A(z)$ only for $z \leq z_F \sim 1/20 \ll \mathcal{O}(1)$

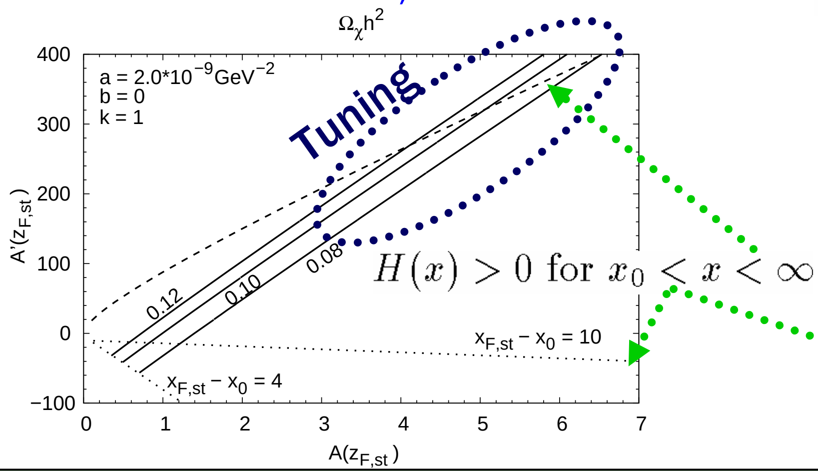
⇒ This suggests a parameterization of $A(z)$ in powers of $(z - z_{F,\mathrm{st}})$:

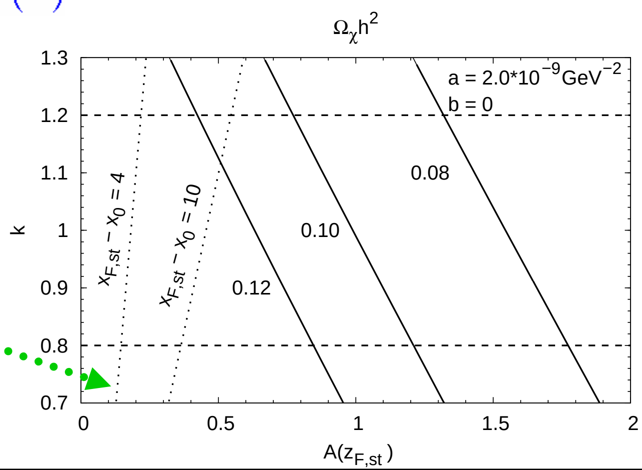
$$A(z) = A(z_{F,\mathrm{st}}) + (z - z_{F,\mathrm{st}})A'(z_{F,\mathrm{st}}) + \frac{1}{2}(z - z_{F,\mathrm{st}})^2 A''(z_{F,\mathrm{st}})$$

subject to the BBN limit: $0.8 \leq k \equiv A(z \to 0) \leq 1.2$

[Olive et al., AP(1999); Lisi et al., PRD(1999); Cyburt et al., AP(2005)]

- Once we know σ, we can constrain $A(z)$:





$\Omega_\chi h^2$ depends on all $H(T_{\mathrm{BBN}} < T < T_F)$ ⇒ Larger allowed region for $H(T_F)$

5. Summary

- Using the CDM relic density we can examine very early universe around $T \sim m_\chi/20 \sim \mathcal{O}(10)\ \mathrm{GeV}$ (well before BBN $T_{\mathrm{BBN}} \sim \mathcal{O}(1)\ \mathrm{MeV}$)

- The relic density of thermal WIMPs depends on the maximal temperature T_0 and on the Hubble parameter $H(T_{\mathrm{BBN}} < T < T_F)$

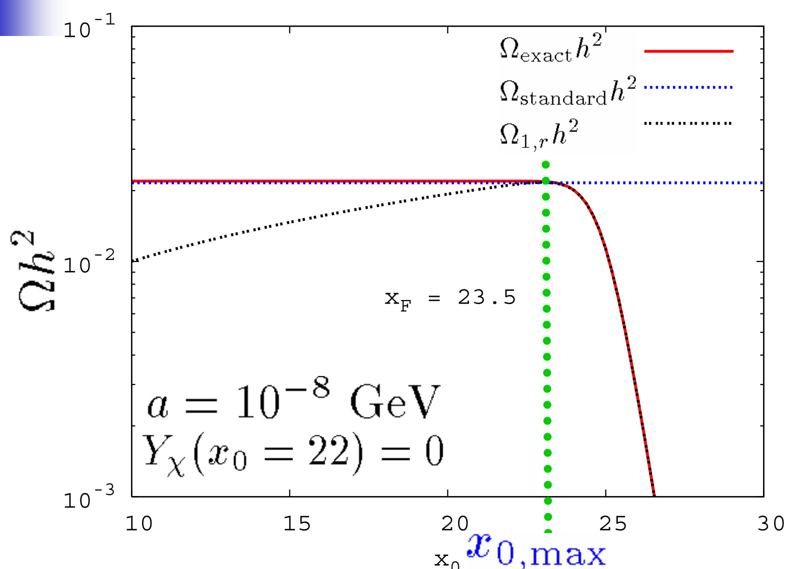
- We derived approximate solutions for the number density which accurately reproduce exact results when full thermal equilibrium is not achieved

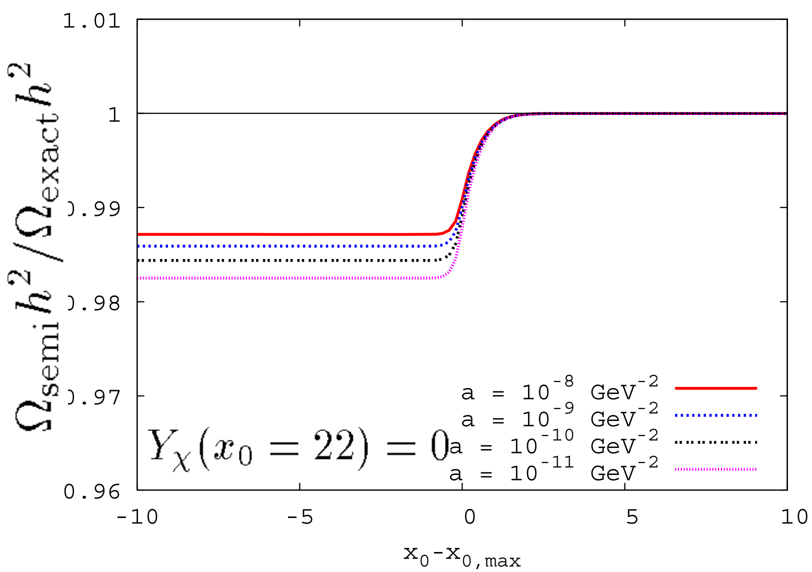
- By applying $\Omega_{\mathrm{CDM}} h^2 = \Omega_{\chi,\mathrm{thermal}} h^2$, we found the lower bound on the maximal temperature: $T_0 > m_\chi/23$

- The sensitivity of $\Omega_{\chi,\mathrm{thermal}} h^2$ on $H(T_F)$ is weak because $\Omega_{\chi,\mathrm{thermal}} h^2$ depends on all $H(T_{\mathrm{BBN}} < T < T_F)$

Backup slides

Semi-analytic solution





- $Y_{1,r}(x_0, x \to \infty)$ $(\propto \Omega_{1,r}h^2)$ has a maximum (left)
- New semi-analytic solution can be constructed: $\Omega_{\rm semi}h^2$ (right)

For $x_0 > x_{0,\rm max}$, use $Y_{1,r}(x_0)$; for $x_0 < x_{0,\rm max}$, use $Y_{1,r}(x_{0,\rm max})$

The semi-analytic solution $\Omega_{\rm semi}h^2$ reproduces the correct final relic density $\Omega_{\rm exact}h^2$ to an accuracy of a few percent